Characterizing Asphalt Mixtures Resistance to Crack Propagation Using the SCB Test:

Weibull Distribution and Entropy Approach

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Acknowledgement

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❖ The main idea of this research came up after a conversation with **Professor Donald Christensen** about Weibull Distribution.

❖ I would like also to Thank my advisors:

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Professor Donald Christensen

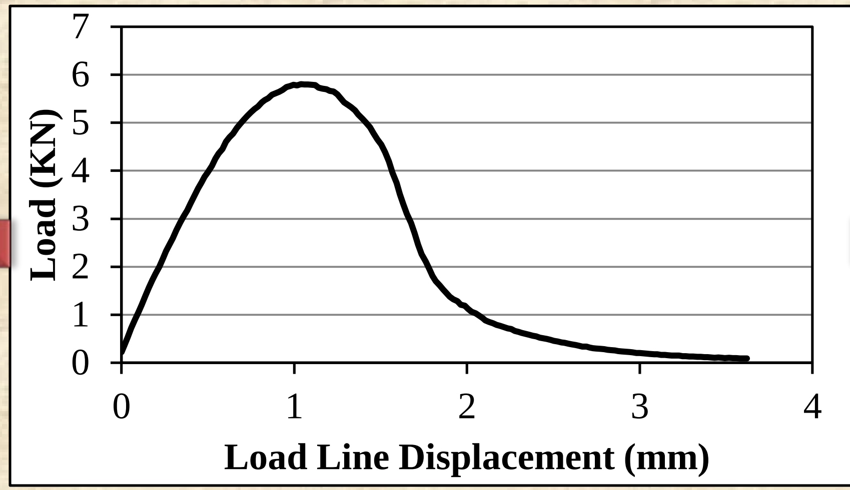
Professor Ramon Bonaquist

Outline

- Introduction and Problem Statement
- Research Objectives
- Fitting Weibull Distribution to SCB Data
- Initial Complex Stiffness Modulus & Shannon Entropy
- Materials and Mix Design
- SCB Testing and New Approach to Analyze the Data
- Validating the New Approach
- Conclusions

Introduction and Problem Statement

SCB Test at Intermediate temperature



$$J_c = \left(\frac{-1}{b}\right)\left(\frac{dU}{da}\right)$$

J_c= critical value of the fracture resistance,
b= sample thickness, a= the notch depth,
U= the strain energy to failure

- Was carried out for elastic and elastic-plastic materials with rounded smooth notch.
- Viscoelastic materials can be treated as non-linear elastic

$$FI = A\frac{G_f}{|m|}$$

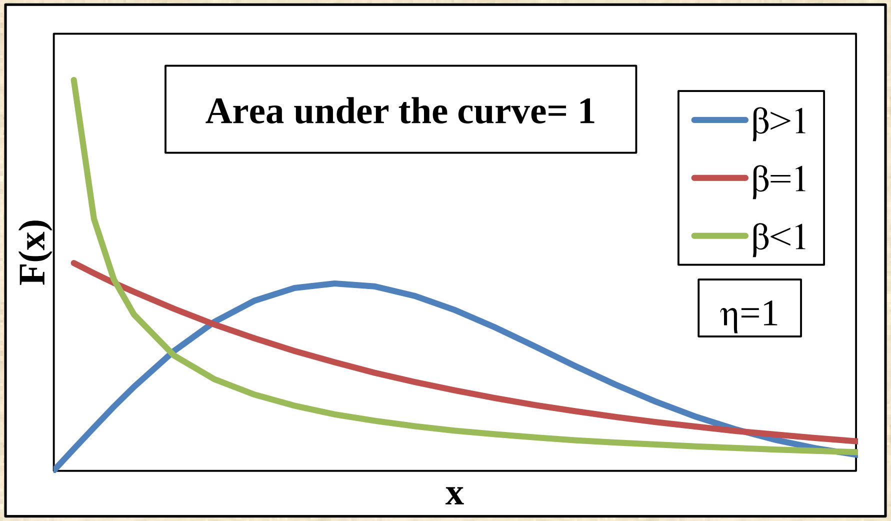
FI= flexibility Index, A=0.01,
G_f= fracture energy,
m= post-peak load slope

- Empirical
- Dissipated energy for asphalt mixtures as viscoelastic materials is unknown during loading.
- Amount of dissipated energy changes by changing temperature and it is not constant for all mixtures.

Weibull Distribution

Probability Density Function (PDF)

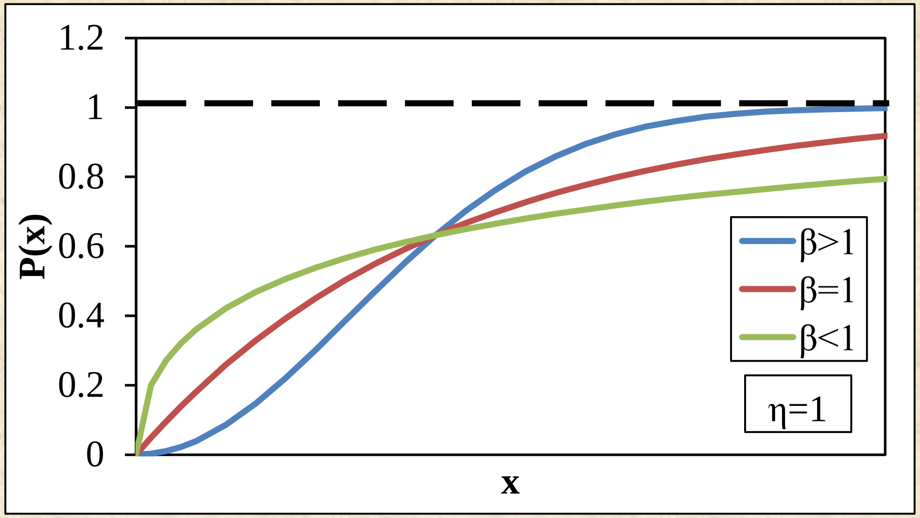
$$f(x) = (x/\eta)^{\beta-1} e^{-(x/\eta)^{\beta}} \beta/\eta$$



X >0

Cumulative Density Function (CDF)

$$P(x) = 1 - e^{-(x/\eta)^{\beta}}$$



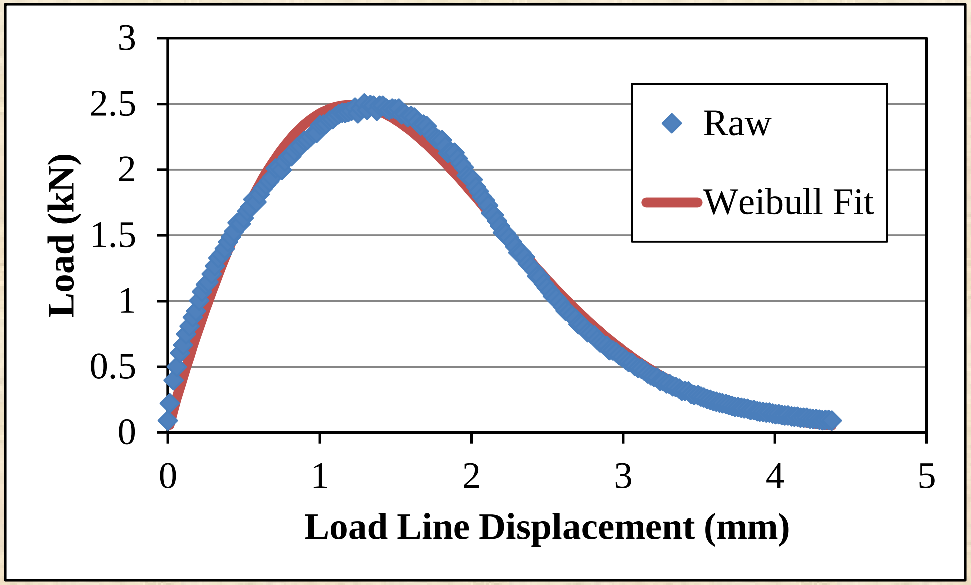
X >0

Research Objectives

1) Fitting Weibull distribution to the relationship between load and load line displacement for SCB test.
2) Deriving mathematical equation for initial complex stiffness modulus of asphalt mixtures “Zo”.
3) Deriving mathematical equation for Shannon entropy “H” (A parameter that represents the mechanical behavior of asphalt mixtures).
4) Develop a new approach to characterize crack propagation resistance of asphalt mixtures using the SCB test.

Fitting Weibull Distribution to SCB Data

$$F = W(x/\eta)^{\beta-1} e^{-(x/\eta)^{\beta}} \beta/\eta$$



F= load (kN),

W= Work of fracture (Joules),

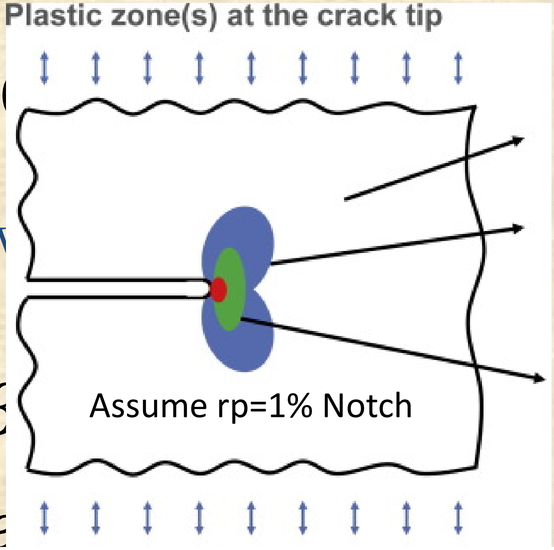
x= Load line displacement (mm),

β= Shape parameter, η= Scale parameter (mm)

Initial Complex Stiffness Modulus

$$\mathrm{F} = \mathrm{W}(x/\eta)^{\beta-1} e^{-(x/\eta)^{\beta}} \beta/\eta$$

$$F/(x)^{\beta-1} = \mathrm{W}(1/\eta)^{\beta-1} e^{-(}$$



Out of more than 200 specimens, the average β value v

deviation was 0.24

$$F/x = (\mathrm{W}/\eta) e^{-(x/\eta)^{\beta}} \beta$$

$$\boxed{(F/\mathrm{A})/(\mathrm{x}/r_p)} = (r_p/\mathrm{A})(\mathrm{W}/\eta)e$$

https://www.sciencedirect.com/science/article/pii/S0142112316302468

Complex Stiffness Modulus of Asphalt Mixtures under indirect tensile stress (Z)

At x =0, the initial modulus can be written as

$$Z_0 = \mathrm{W} r_p \beta / \mathrm{A}\eta^2$$

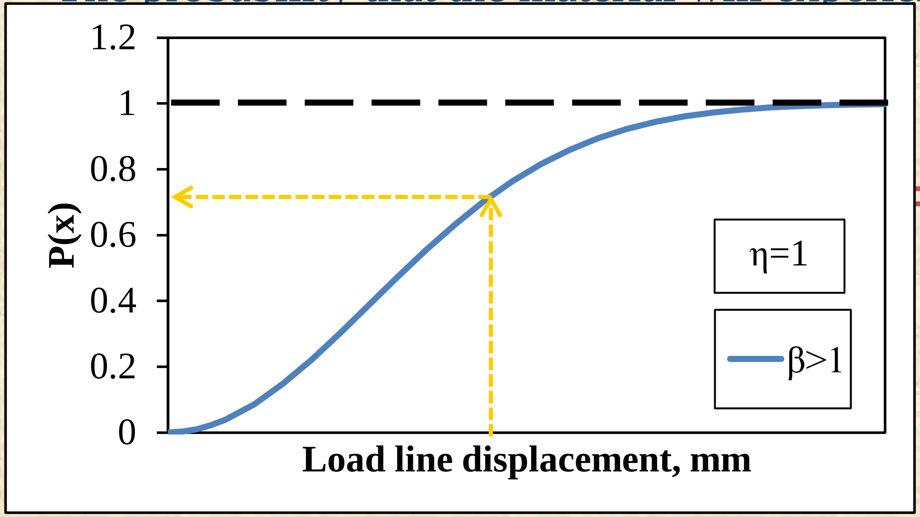
Introduction to Shannon Entropy

$$Z = (r_p/\mathrm{A})(\mathrm{W}/\eta)e^{-(x/\eta)^{\beta}}\beta/\eta$$

$$Z_0 = \mathrm{W}r_p\beta/\mathrm{A}\eta^2$$

$$Z/Z_0 = e^{-(x/\eta)^{\beta}}$$

The probability that the material will experience total failure "Percentage of drop in value (x) can be expressed as:



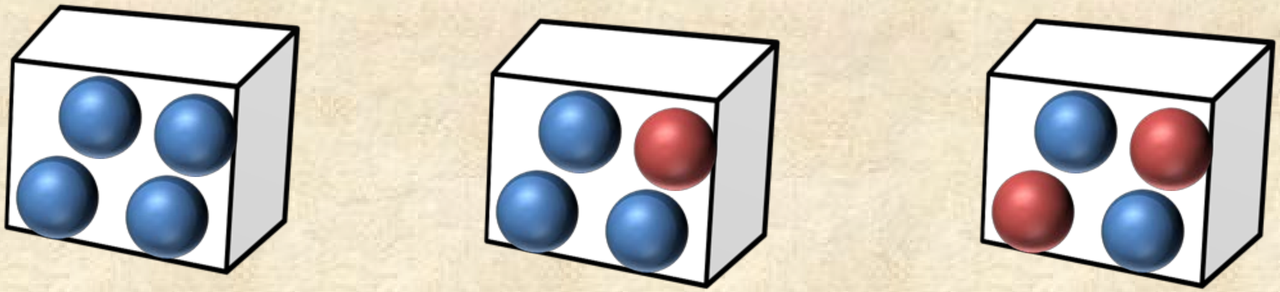
$$= \left[1 - e^{-(x/\eta)^{\beta}}\right]$$

Cumulative Density function of Weibull Distribution β>1

Entropy & Shannon Entropy

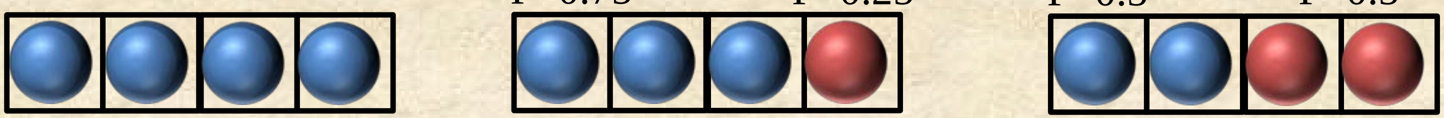
- Entropy is a physical property that indicates the molecular state of a system (a measure of disorder).
- The mechanical (effective) work of a system is a function of its entropy and internal energy.
- In statistical mechanics, Shannon entropy can be used as an indication to the mechanical behavior of a system with unknown state.
- Physical entropy can be estimated from Shannon entropy by multiplying it by a constant.

Shannon Entropy



We want to pick a random ball from each box and return it back for four times

P=1 P=0.75 P=0.25 P=0.5 P=0.5



What is the probability that the choice will match what is in the box?

P=1*1*1*1= 1 P=0.75*0.75*0.75*0.25= 0.1 P=0.5*0.5*0.5*0.5= 0.06

Probability (Knowledge)	**High**	**Medium**	**Low**

$$Shannon \text{ Entropy H} = -\sum P_i log_b P_i$$

$H = -[1 * log_{10} 1] = 0$

$H = -[0.75 * log_{10} 0.75 + 0.25 * log_{10} 0.25] = 0.244$

$H = -[0.5 * log_{10} 0.5 + 0.5 * log_{10} 0.5] = 0.3$

Shannon Entropy	**Low**	**Medium**	**High**

Shannon Entropy

$$\tilde{f}(\tilde{x}) = r_p \mathrm{f}(\tilde{x}) \qquad H = -\int_{-\infty}^{\infty} \tilde{f}(\tilde{x}) ln\left(\tilde{f}(\tilde{x})\right) d\tilde{x} \qquad \tilde{x} = x/r_p$$

Euler-Mascheroni constant (0.577) $= \gamma\left(1 - \frac{1}{\beta}\right) + \ln\left(\frac{\lambda}{\beta}\right) + 1 \qquad \lambda = \eta/r_p$

Properties of Shannon Entropy:

- A unique number for each distribution (SCB sample) depending on β and λ.
- This unique number can be used as an indication to the mechanical properties (viscoelasticity) of mixtures.

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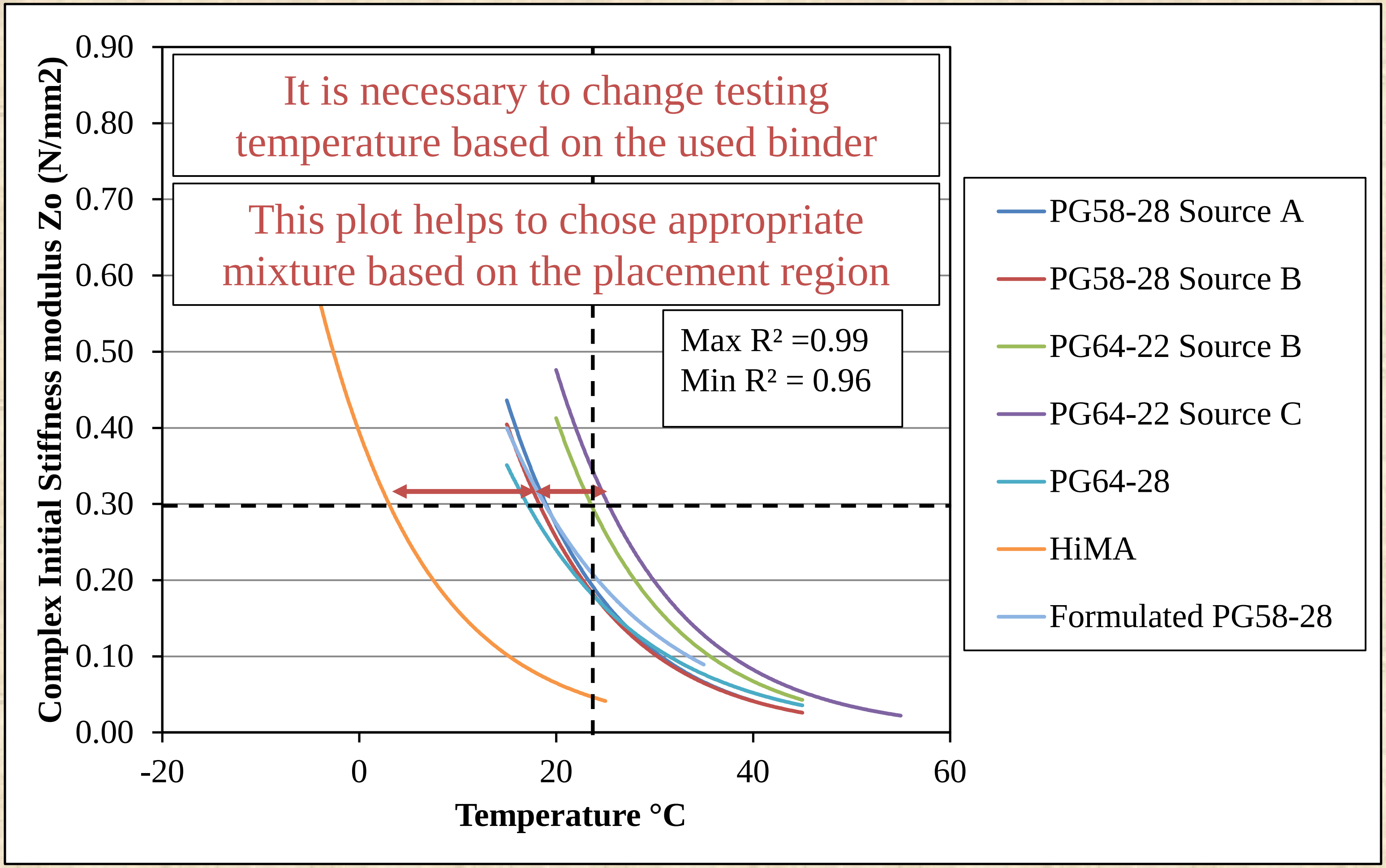
Materials and Mix Design

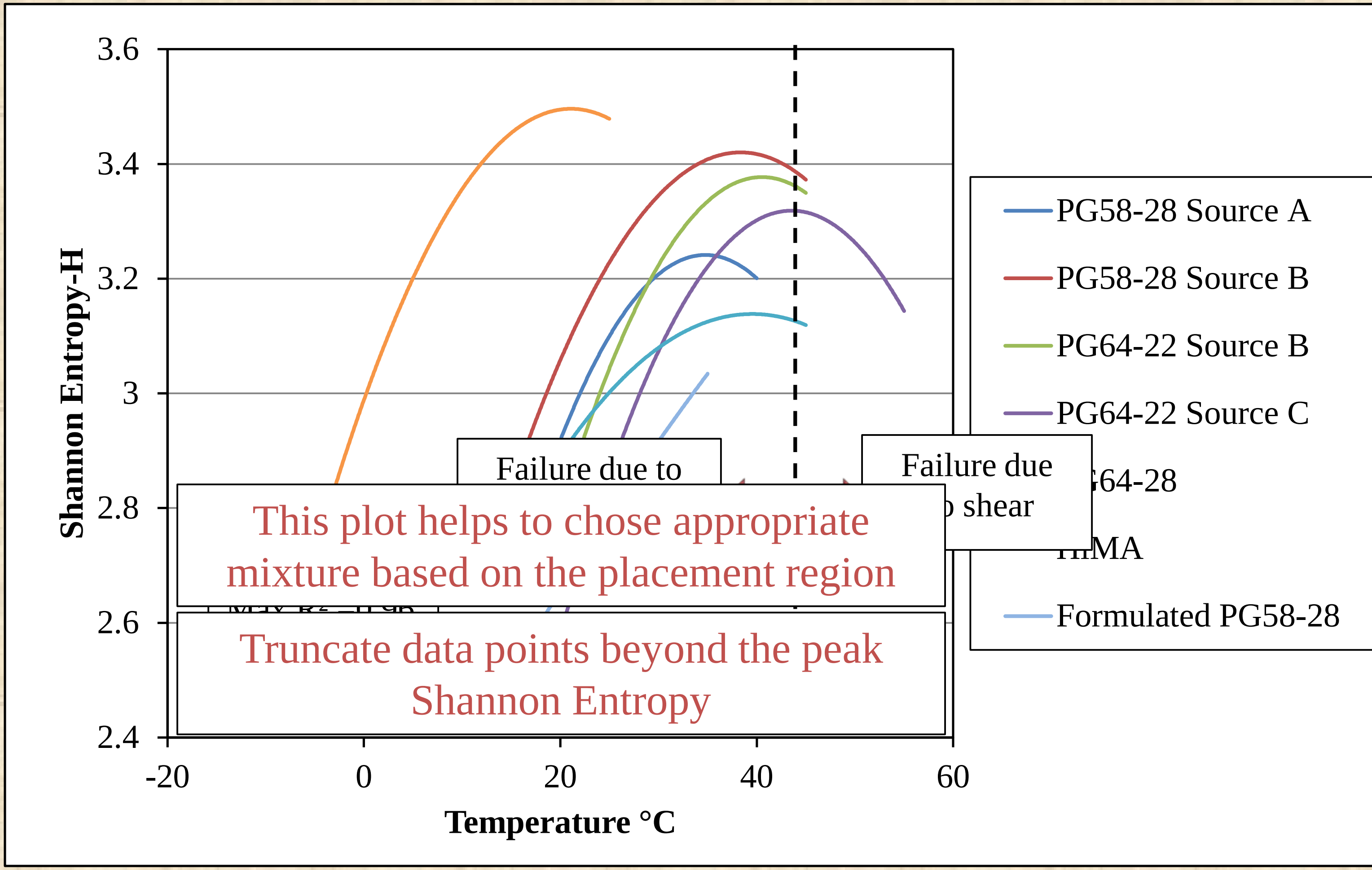
Sieve Size (mm)	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	Binder Content (%)
% Passing by Weight	100	98	85	58	42	27	15	9	6	6.5

Incorporated Binder	Temperature ºC										
	-15	-5	5	10	15	20	25	30	35	40	45
PG64-22 (2 Sources)					✓	✓	✓	✓	✓	✓	✓
PG 58-28 (2 Sources)			✓	✓	✓	✓	✓	✓	✓	✓	
PG 64-28			✓	✓	✓		✓	✓	✓		✓
HiMA	✓	✓	✓		✓		✓				
Formulated PG58-28			✓	✓	✓		✓	✓	✓		

SCB Testing and Analysis



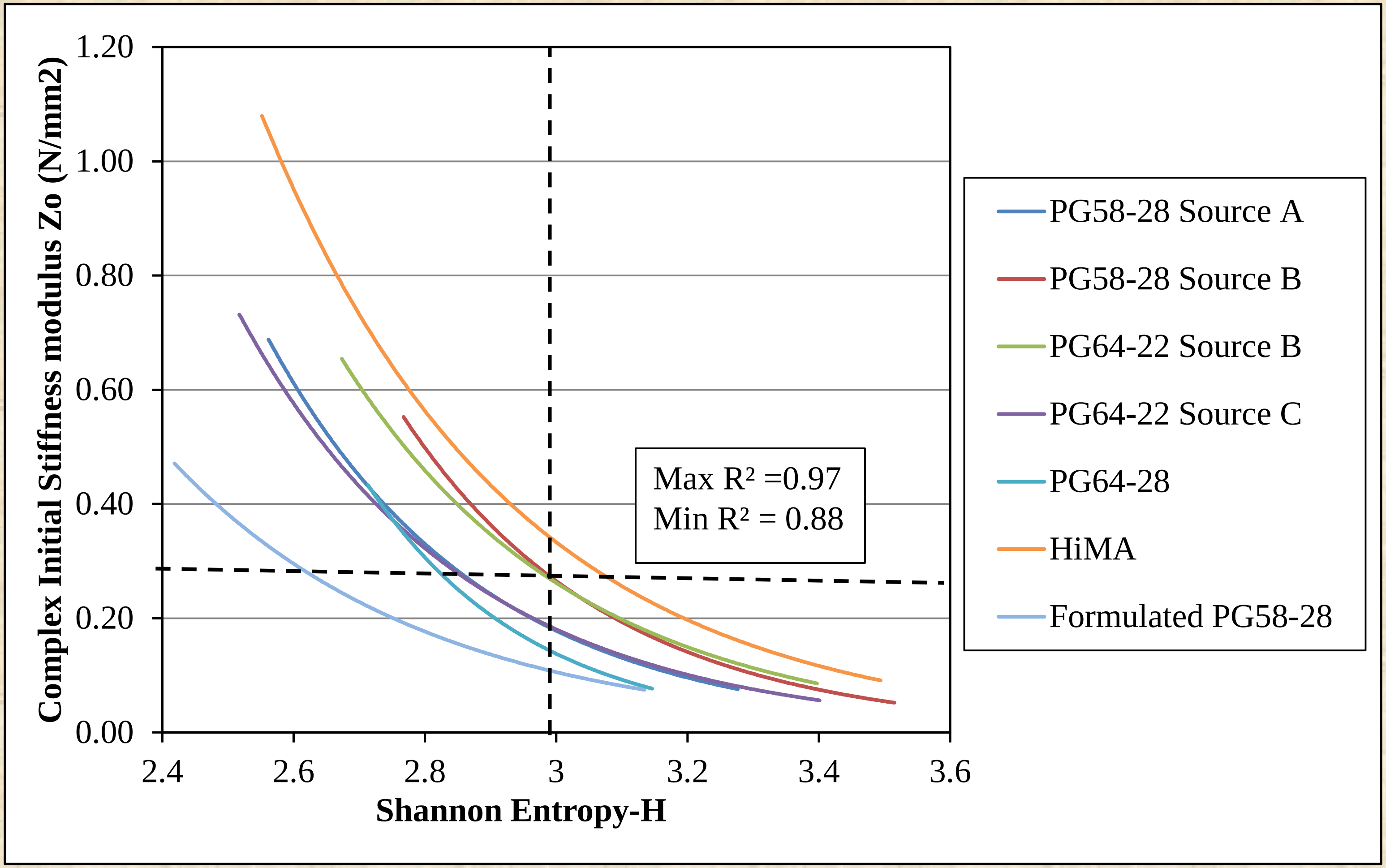
SCB Testing and Analysis



This plot helps to chose appropriate mixture based on the placement region

Truncate data points beyond the peak Shannon Entropy

SCB Testing and Analysis



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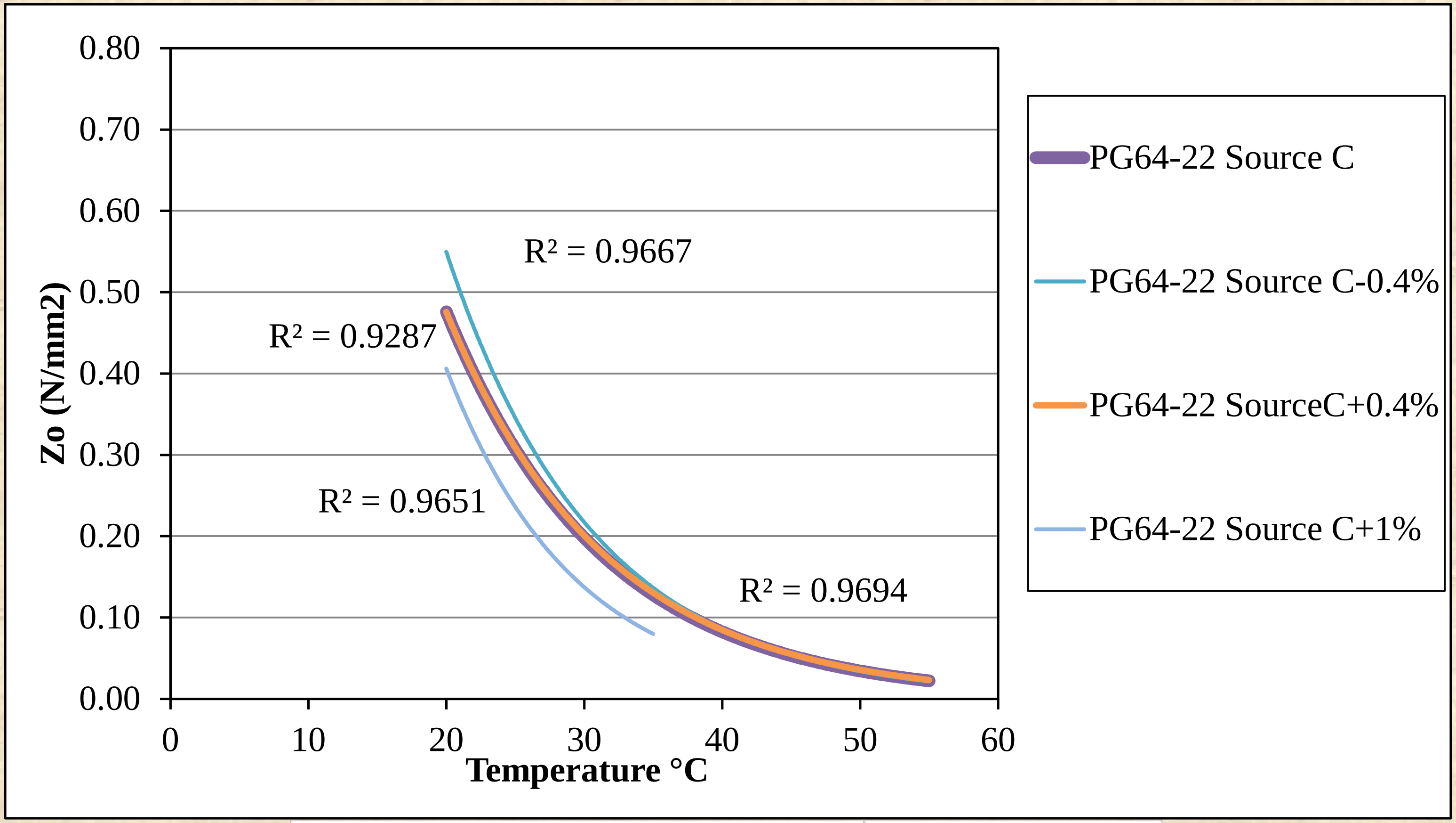
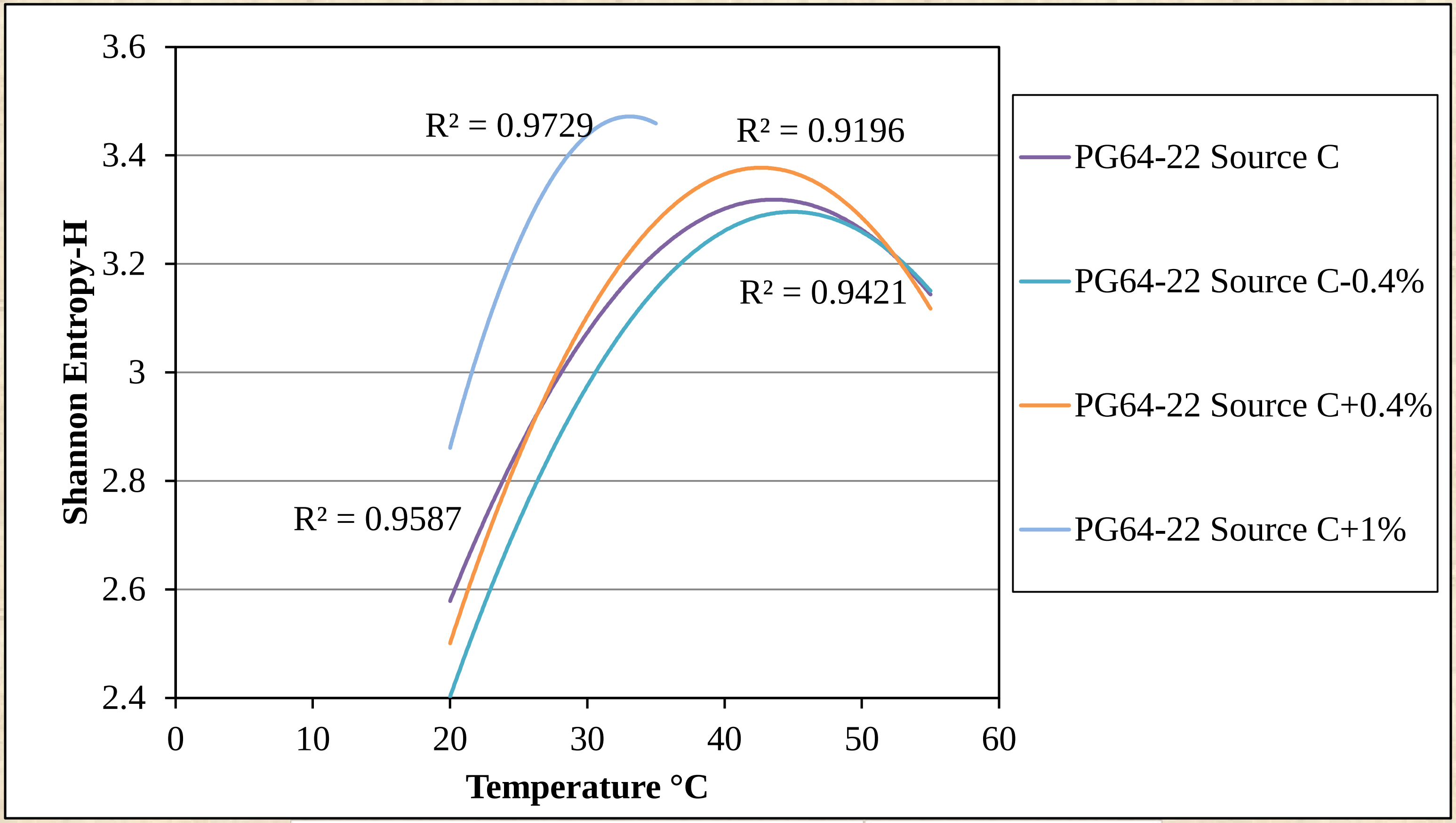
Validating the New Methodology

❖ Binder Content



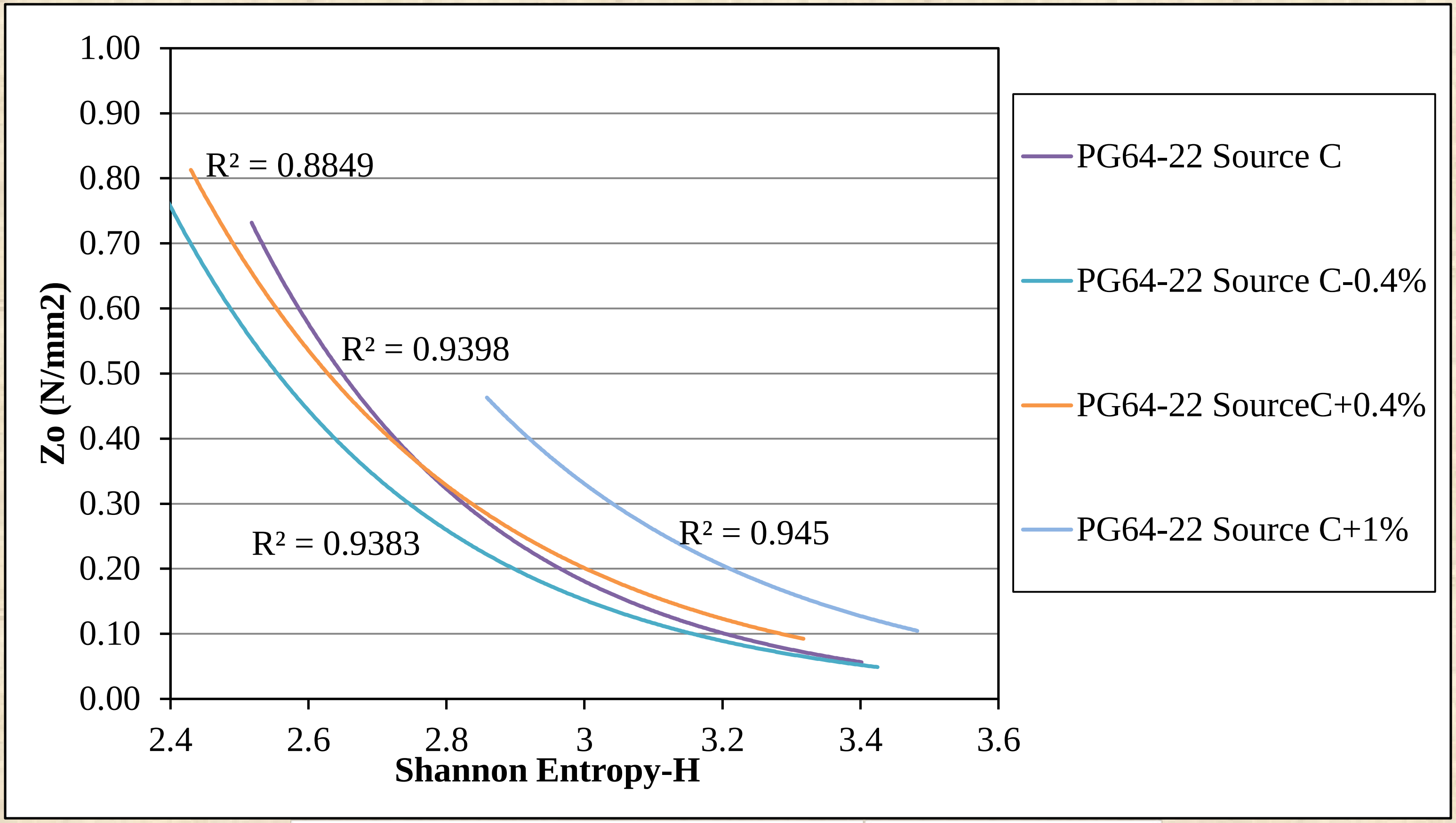
Validating the New Methodology

- Binder Content



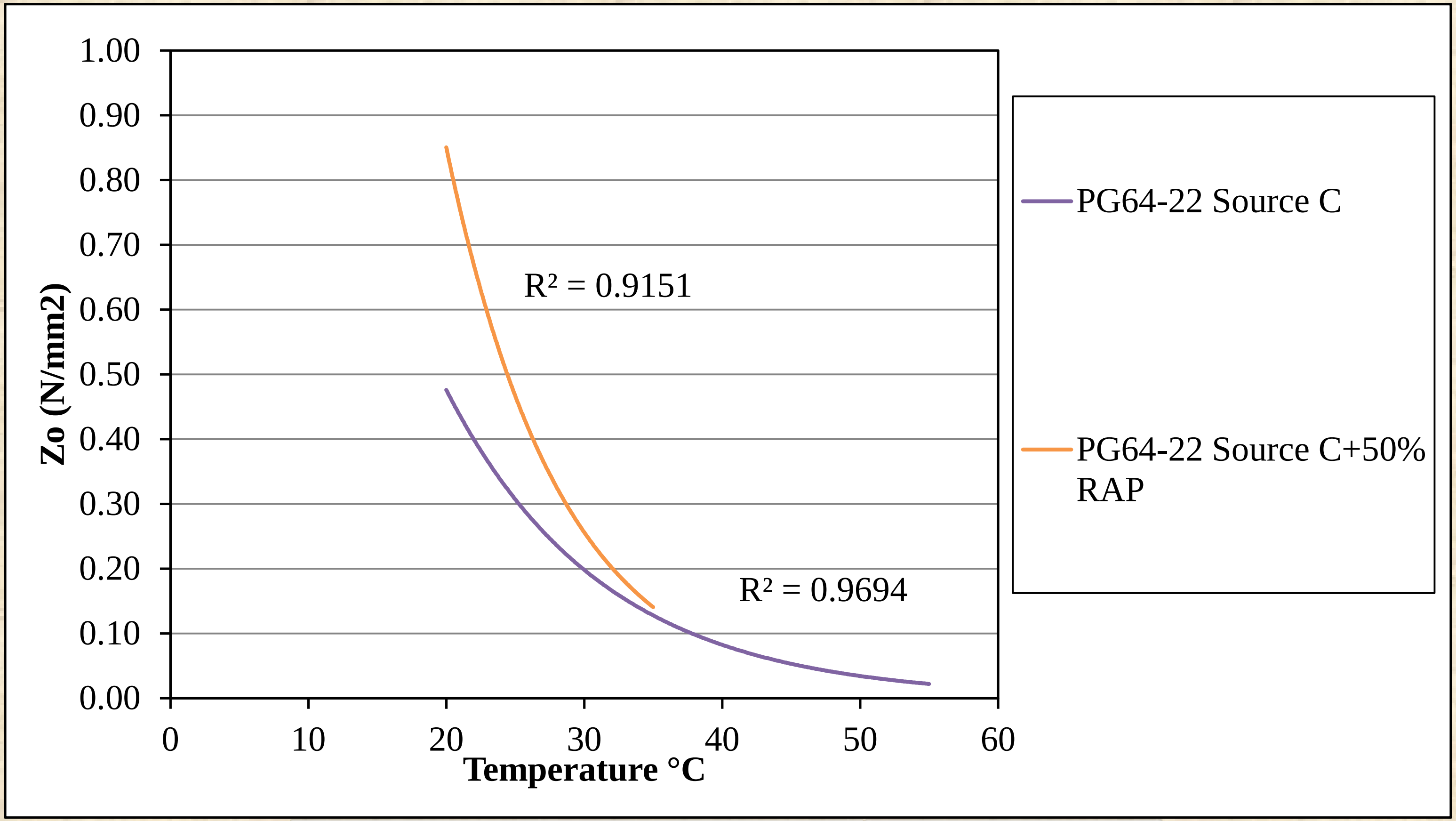
Validating the New Methodology

- Binder Content



Validating the New Methodology

- RAP Content



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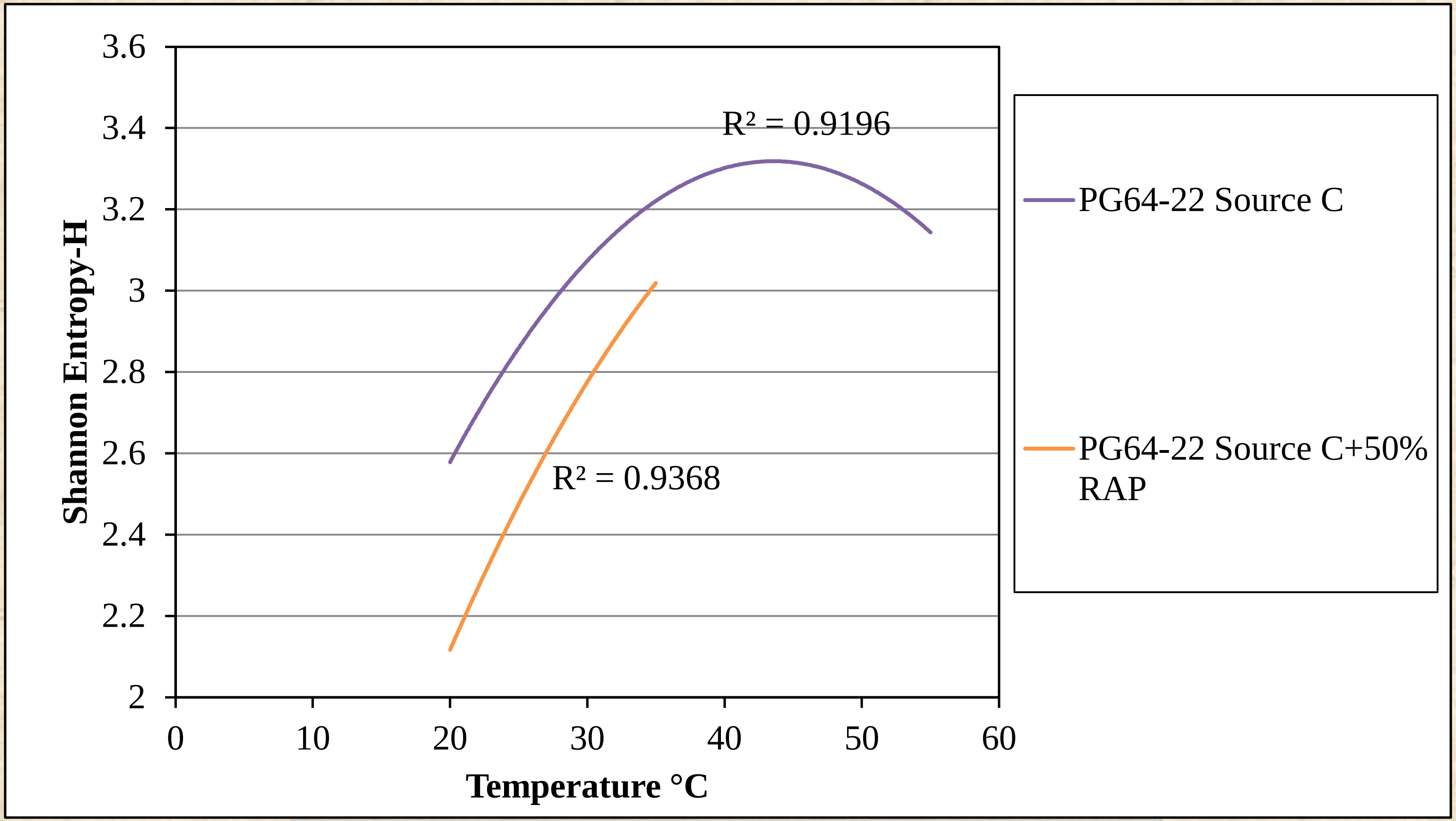
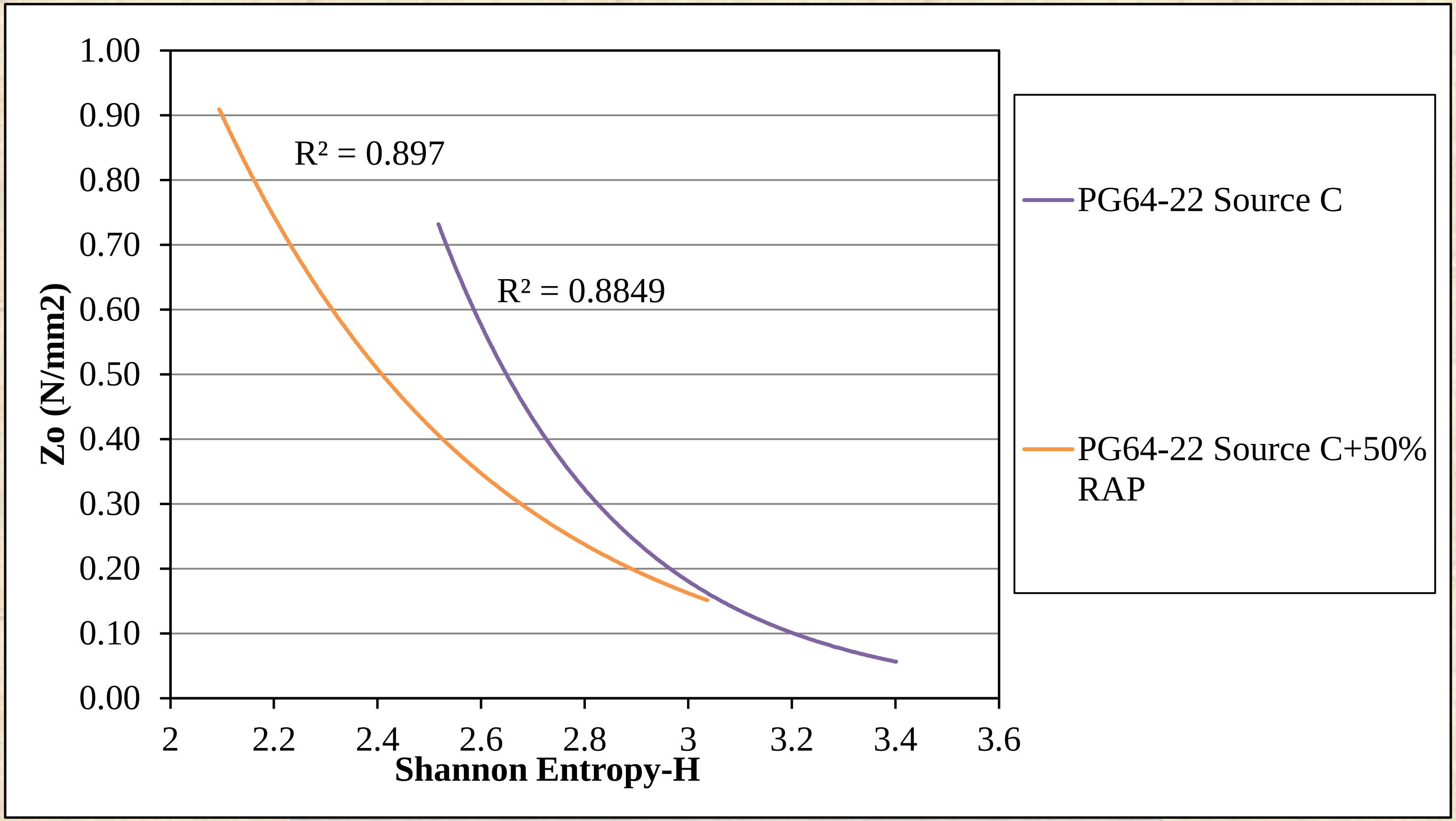
Validating the New Methodology

❖ RAP Content



Validating the New Methodology

- RAP Content



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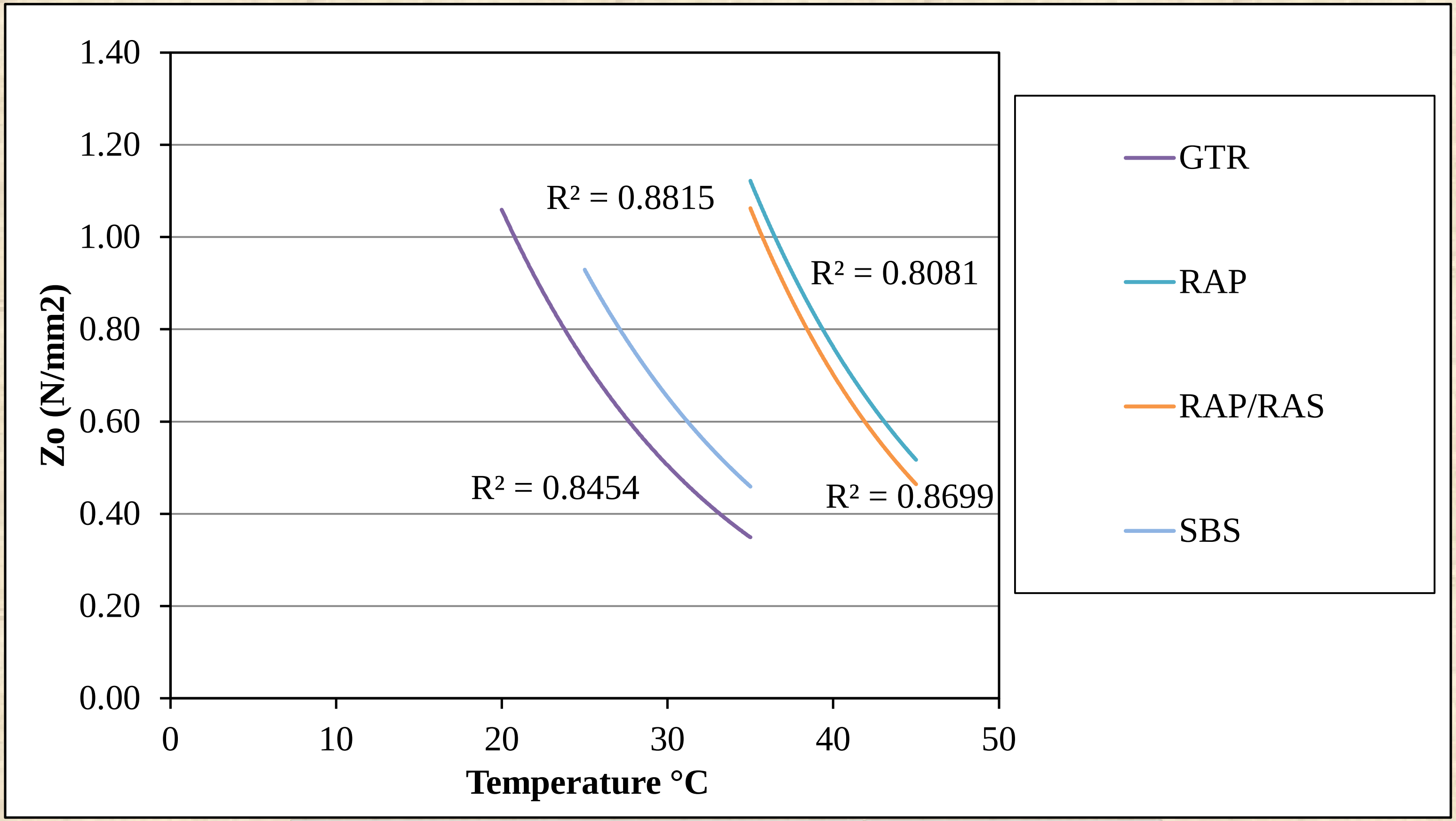
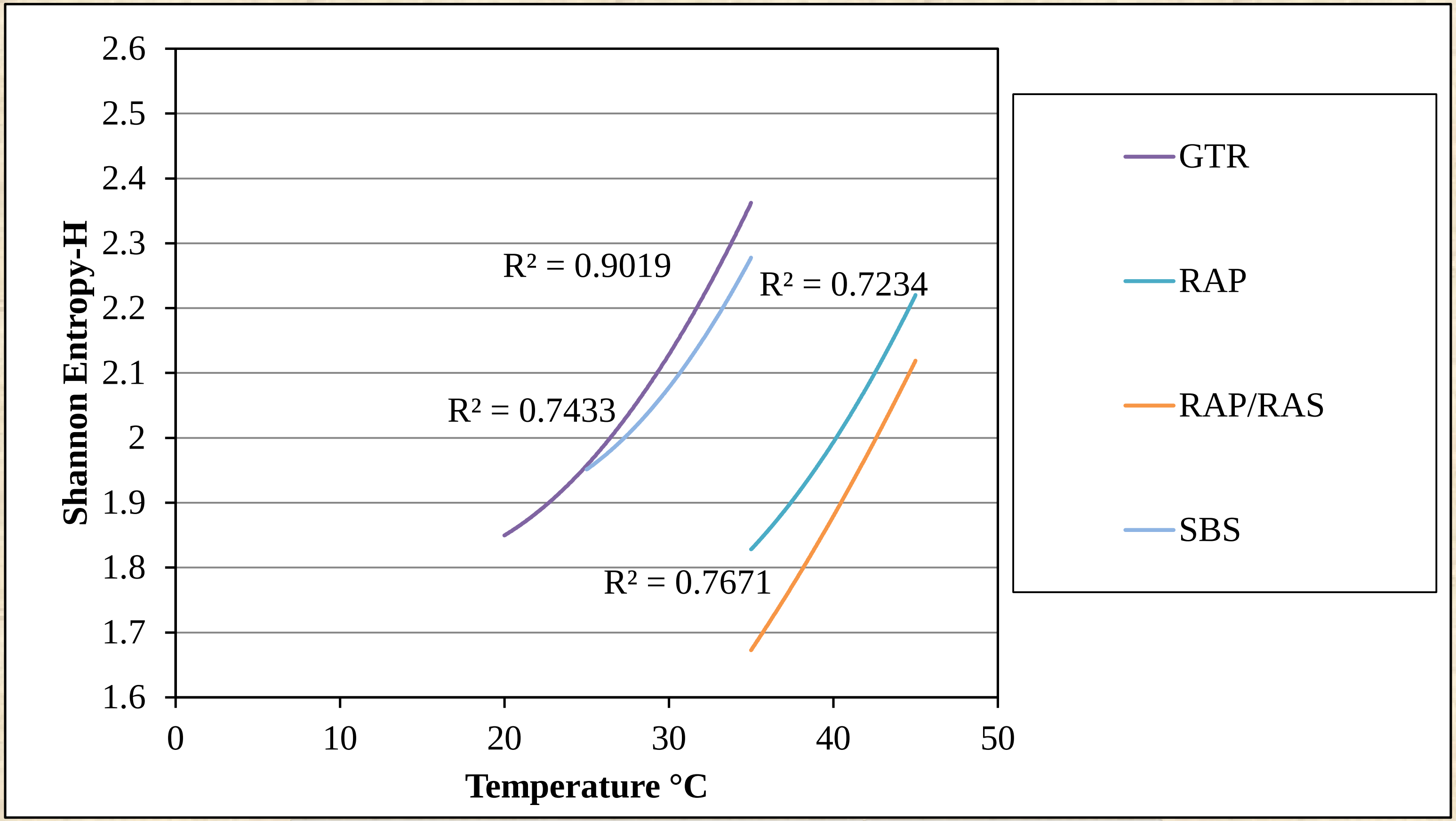
Validating the New Methodology

- NCAT ALF Mix



Validating the New Methodology

❖ NCAT ALF Mix



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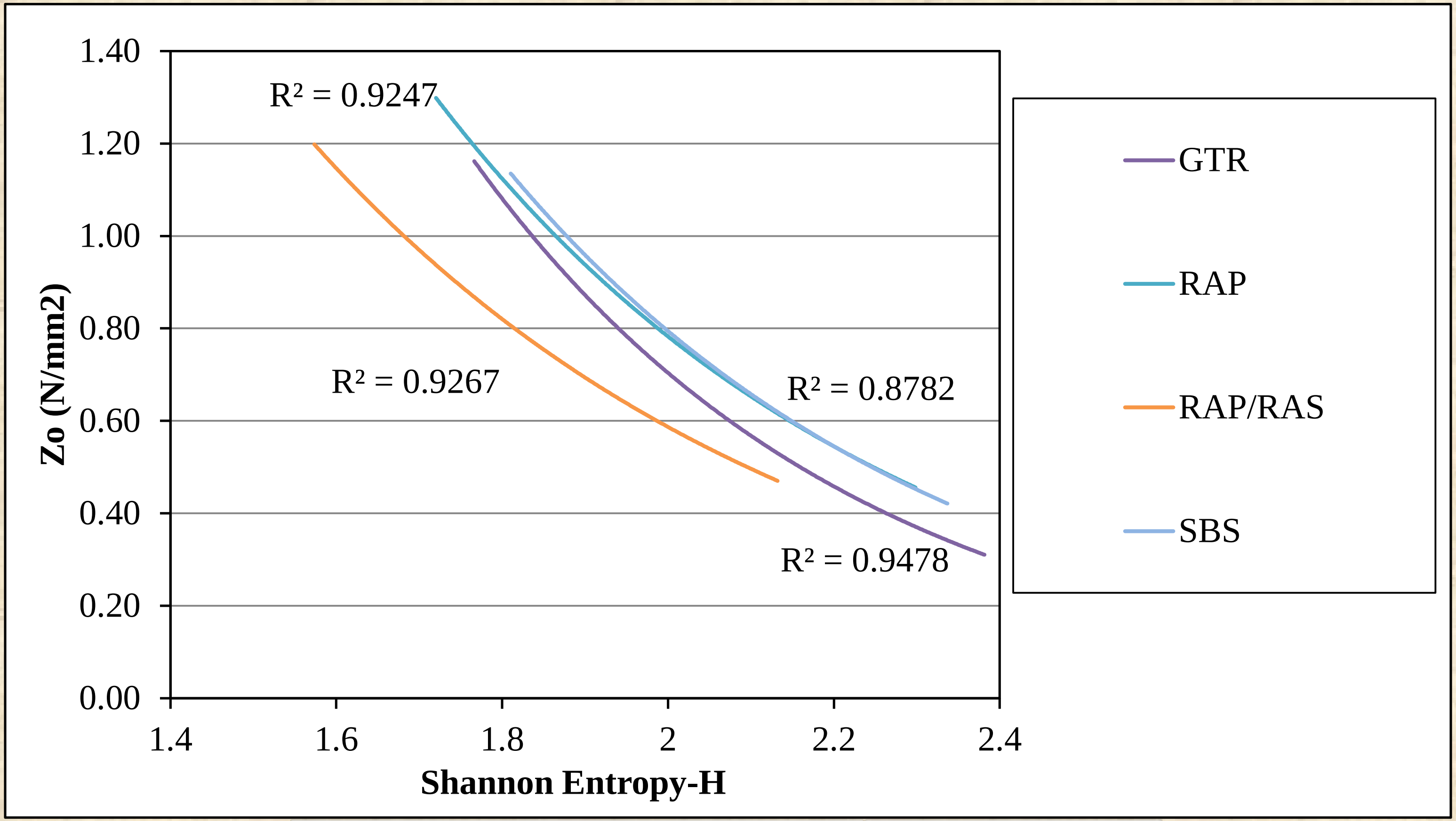
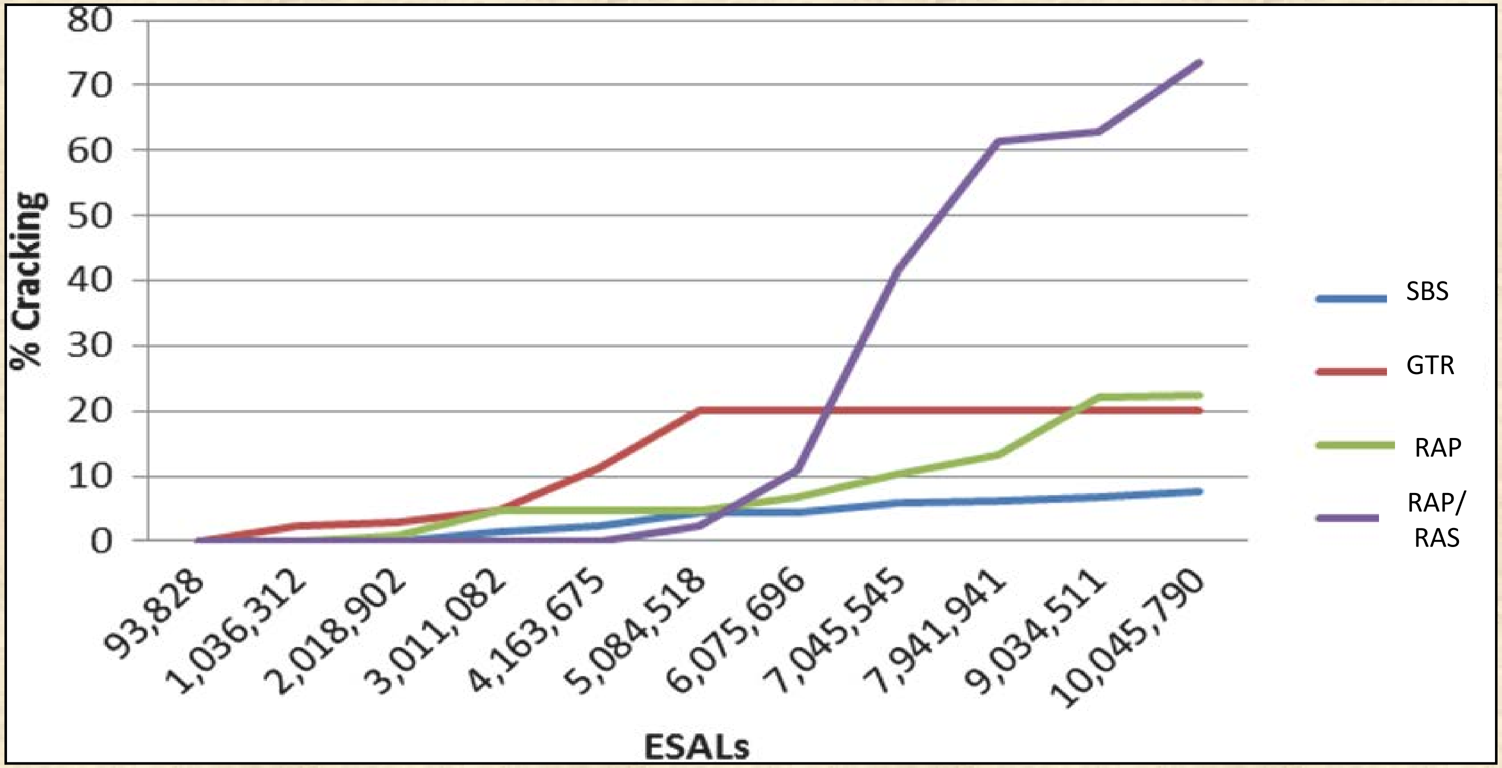
Validating the New Methodology

- NCAT ALF Mix



Validating the New Methodology

❖ NCAT ALF Mix



Laboratory and Field Evaluation of Florida Mixtures at the 2012 National Center for Asphalt Technology Pavement Test Track

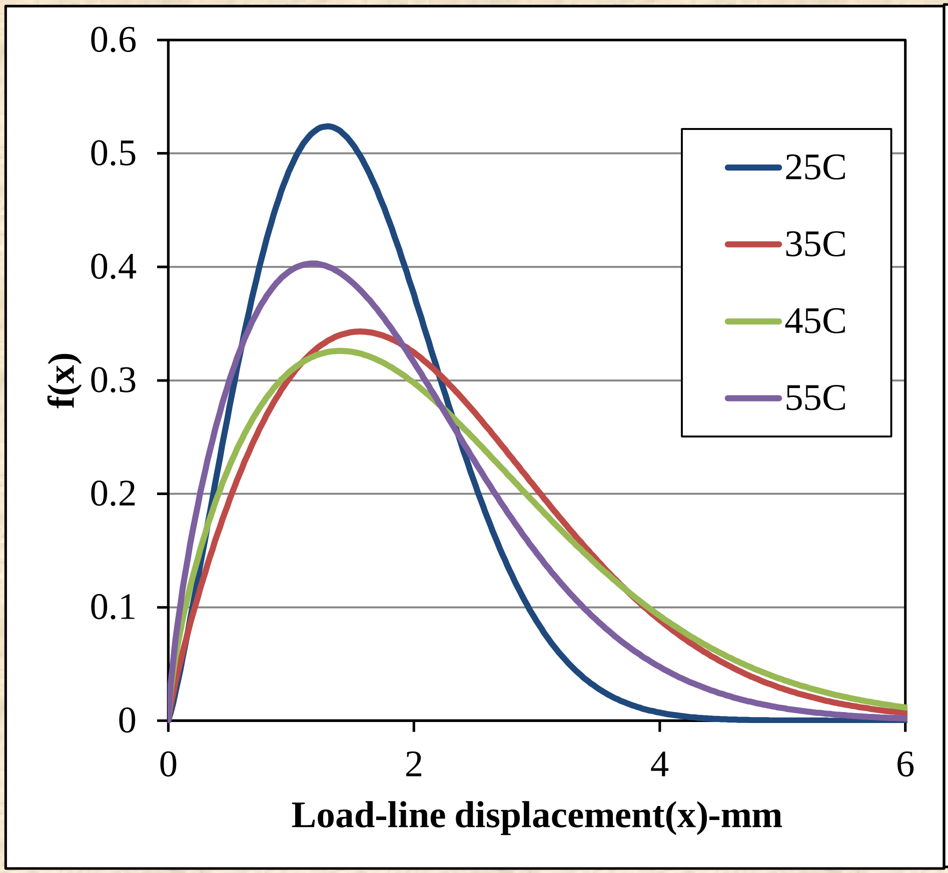
J. Richard Willis, Adam J. Taylor, and Tanya M. Nash

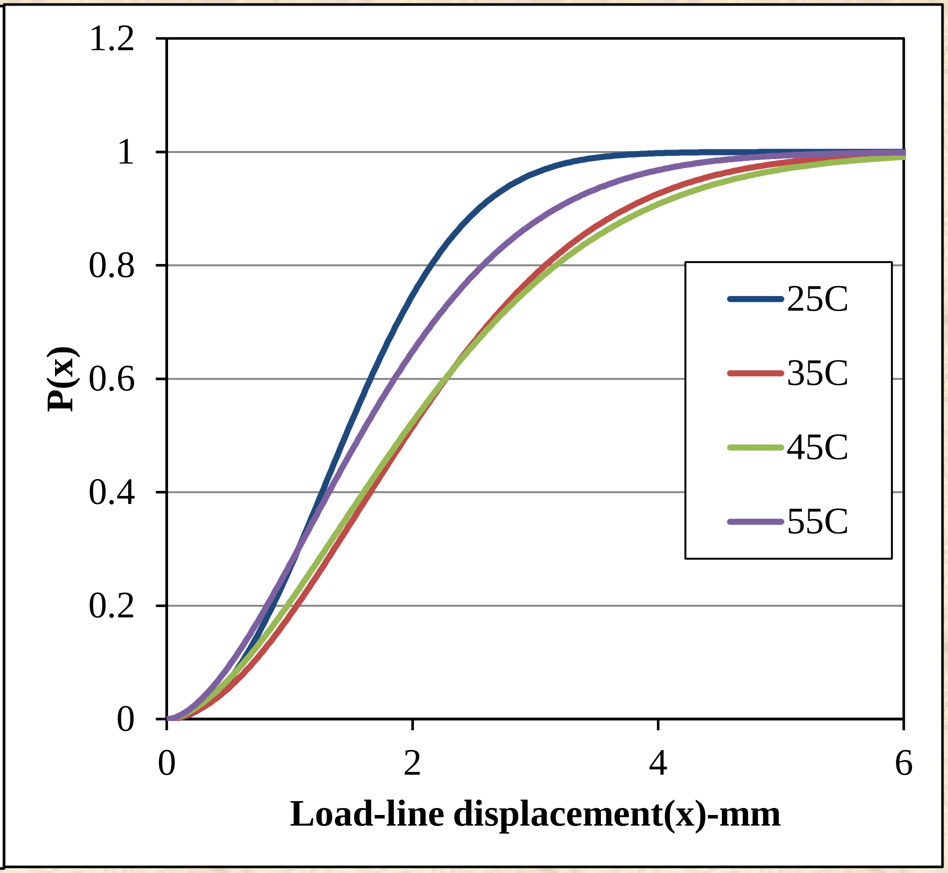
Conclusions

- Weibull distribution can be used to fit crack propagation data from the SCB test.
- The initial complex stiffness modulus and Shannon entropy (a measure of the mechanical behavior) can be derived from the Weibull fitted distributions.
- Correlations between testing temperature and initial complex stiffness modulus and Shannon entropy are useful to choose appropriate mixture based on the placement region.
- Based on this study, asphalt mixtures should be compared at the same state (Shannon entropy value), or at the same initial complex stiffness modulus. This might require testing at multiple temperatures.

THANK YOU!

SCB Testing and Analysis





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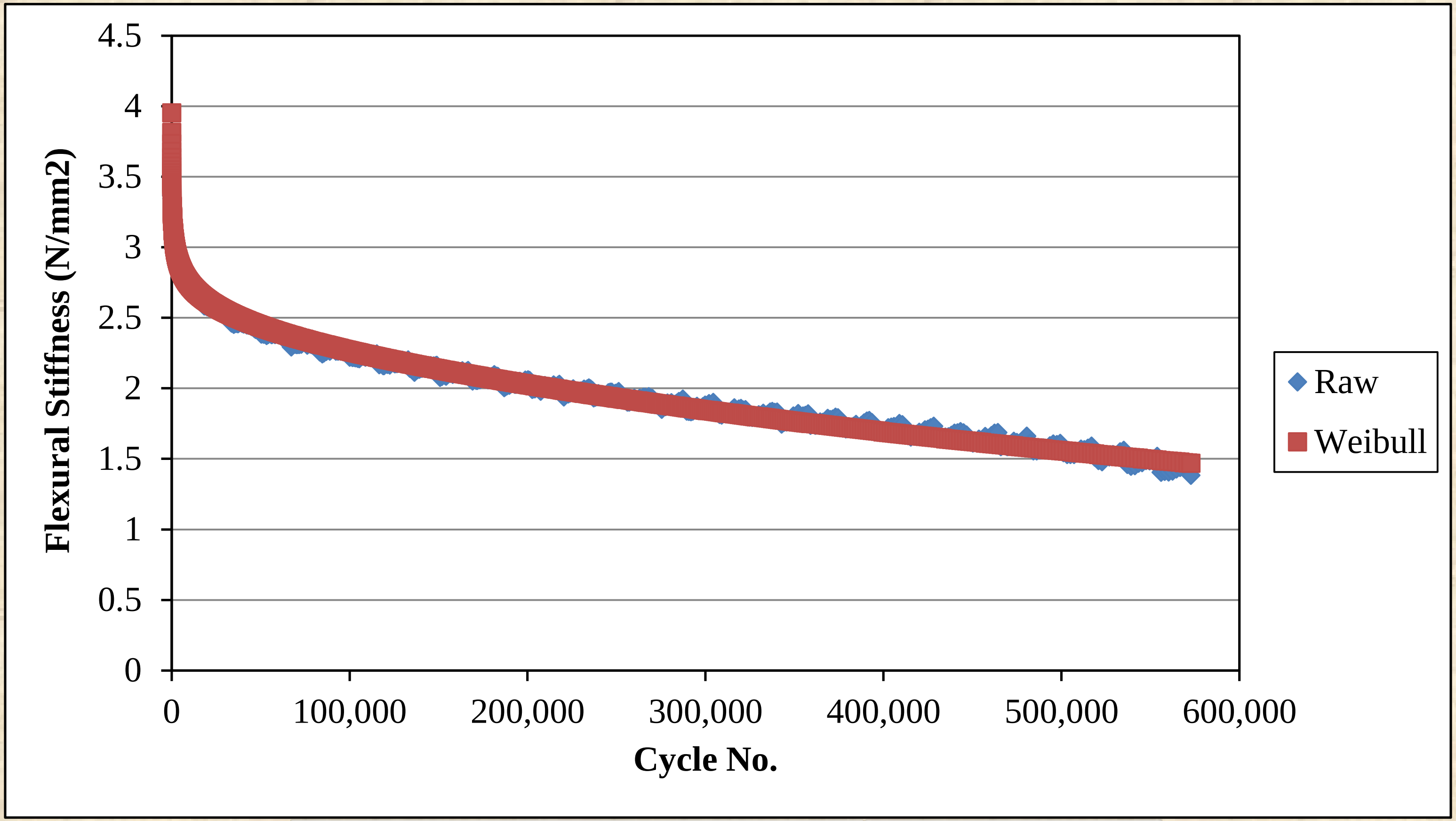
What is Next?

- Initial complex stiffness modulus and Shannon entropy can be predicted for other mixture and binder tests using similar approach.
- Initial complex stiffness modulus from different tests can be correlated using basics mechanics of materials.
- Shannon entropy from different tests might be correlated.
- A master curve can be developed from different tests with different modes of failures and used in pavement design.

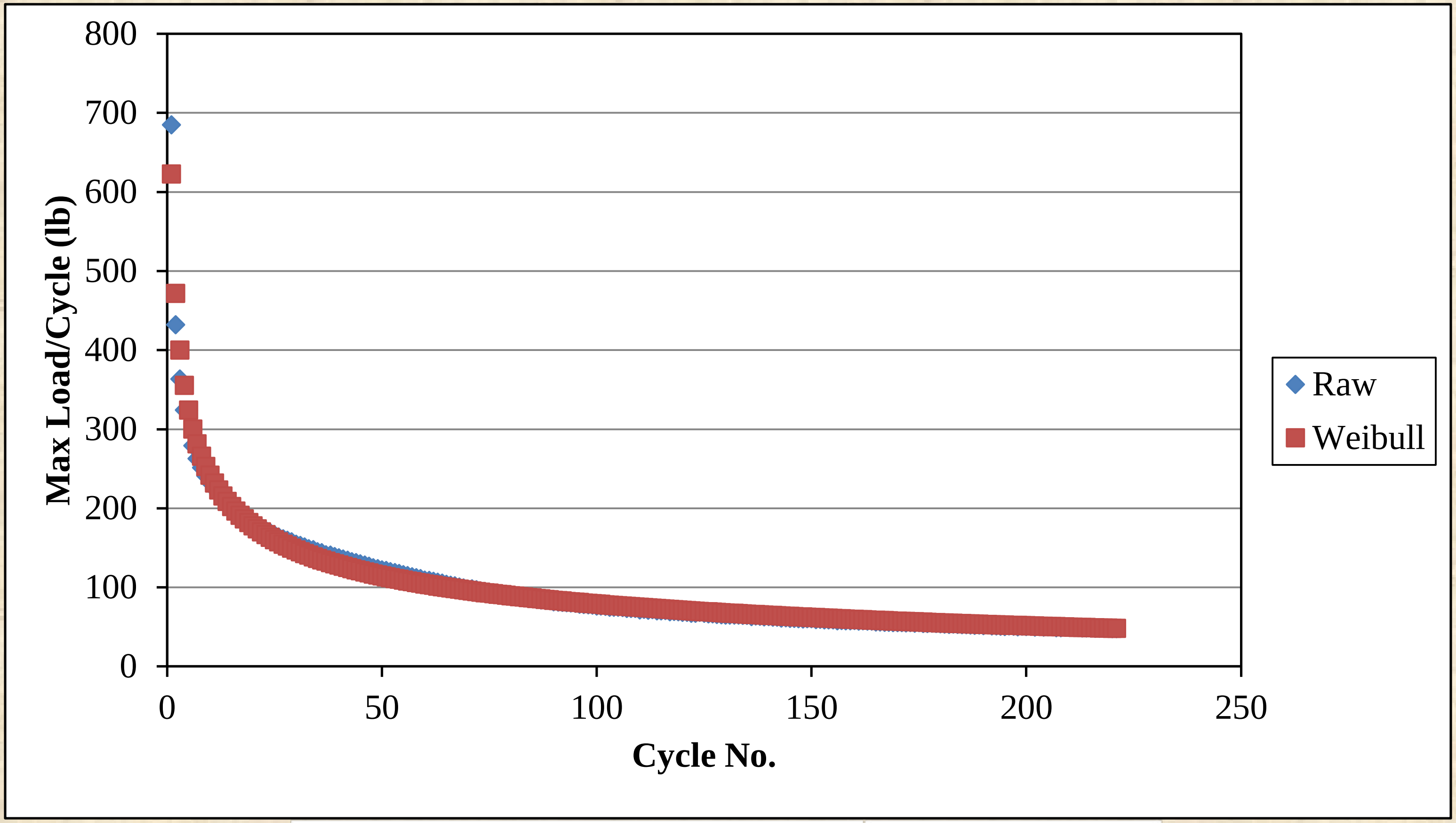
What is Next?

- Beam Fatigue



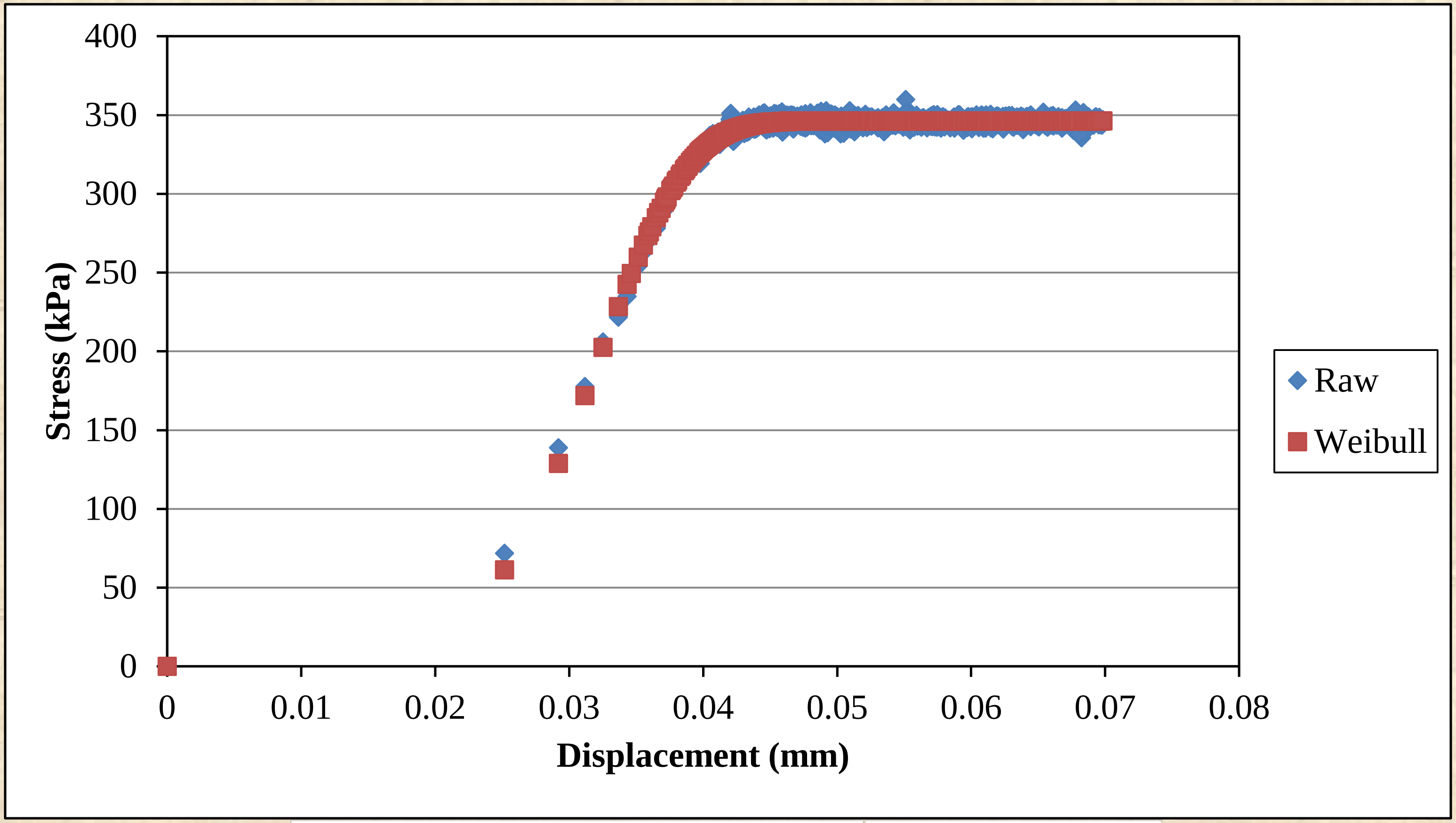
What is Next?

- Texas Overlay



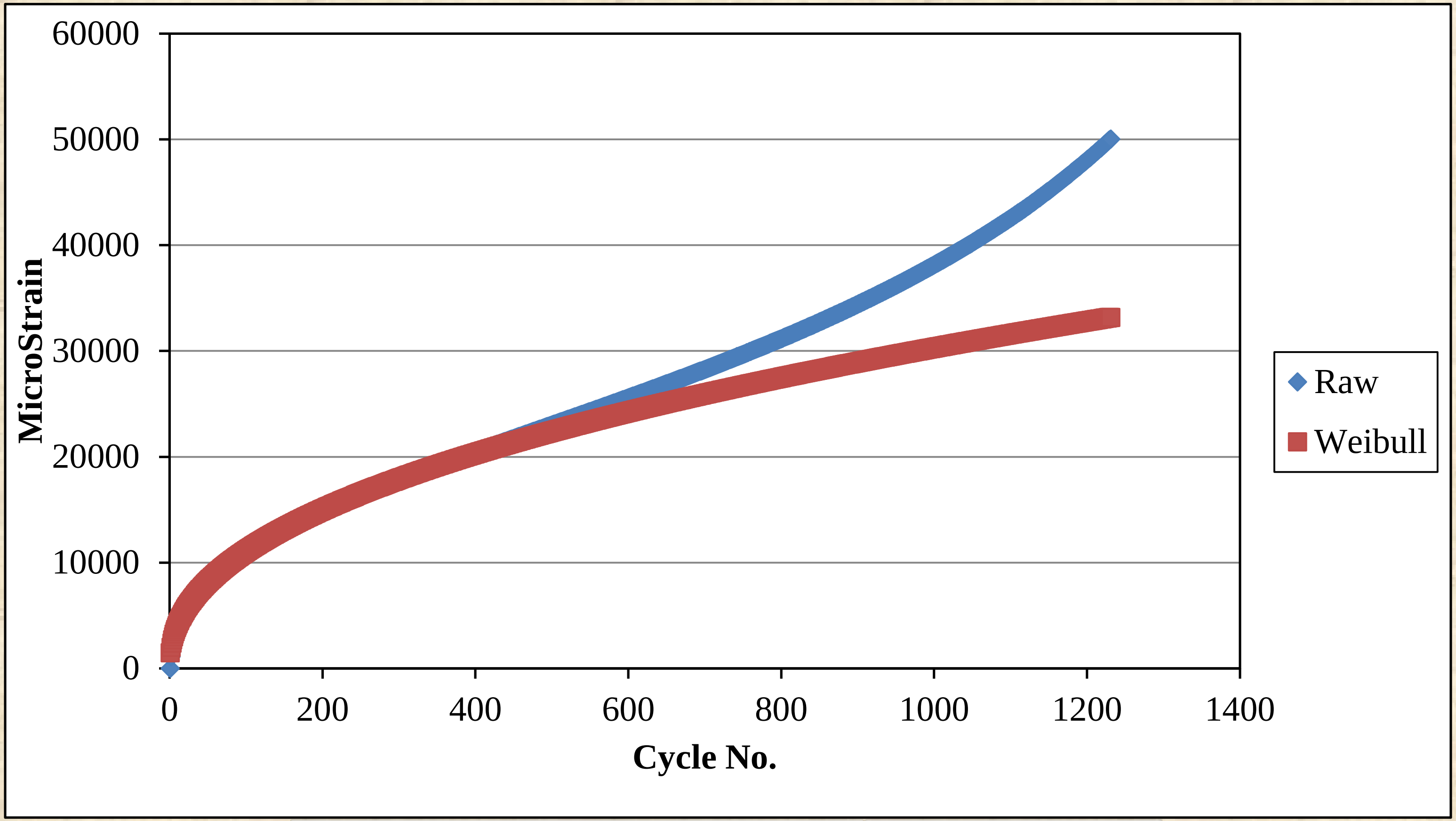
What is Next?

- Cyclic tension (Pull-Pull)



What is Next?

❖ Flow Number



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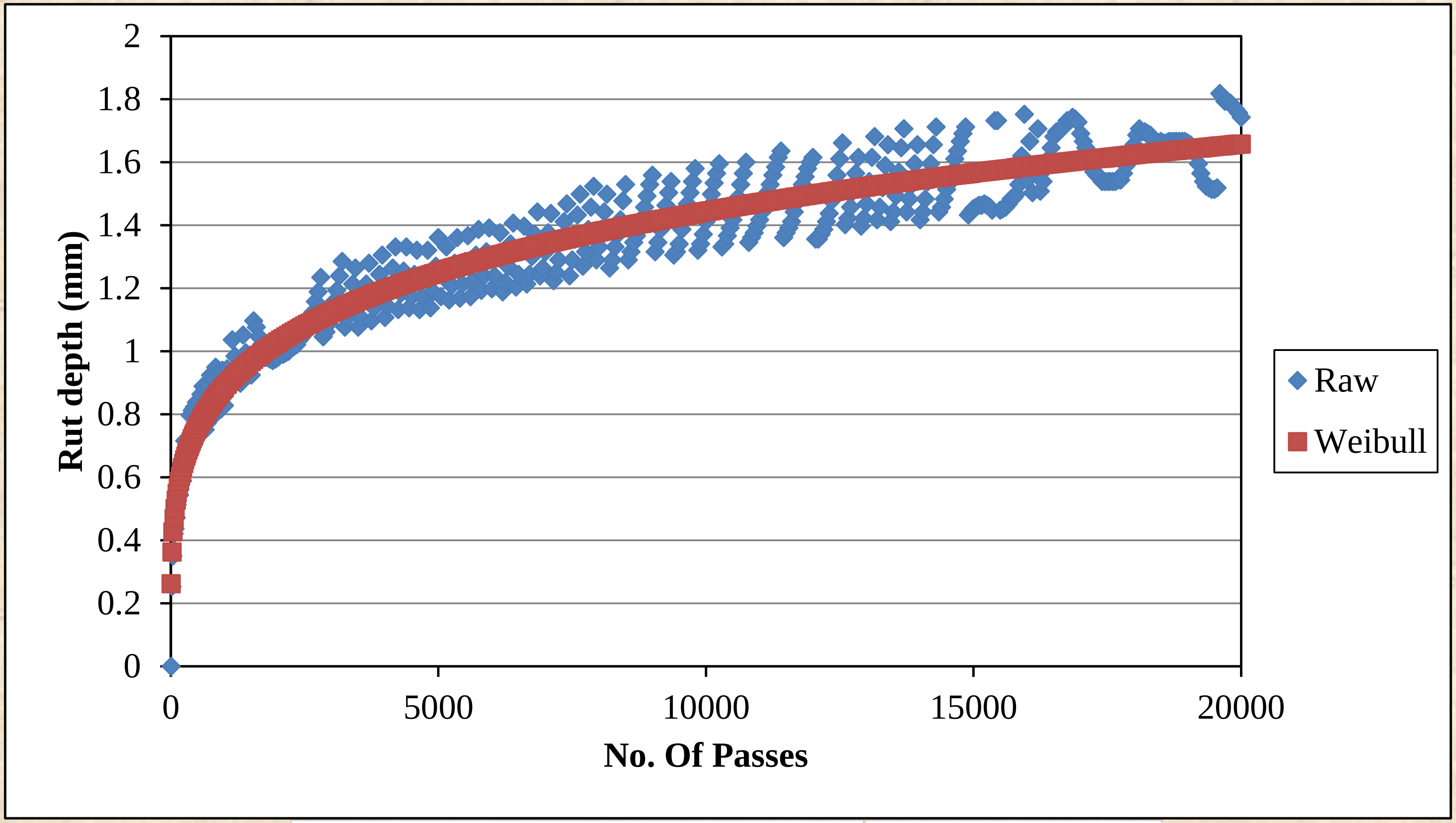
What is Next?

- HWTD



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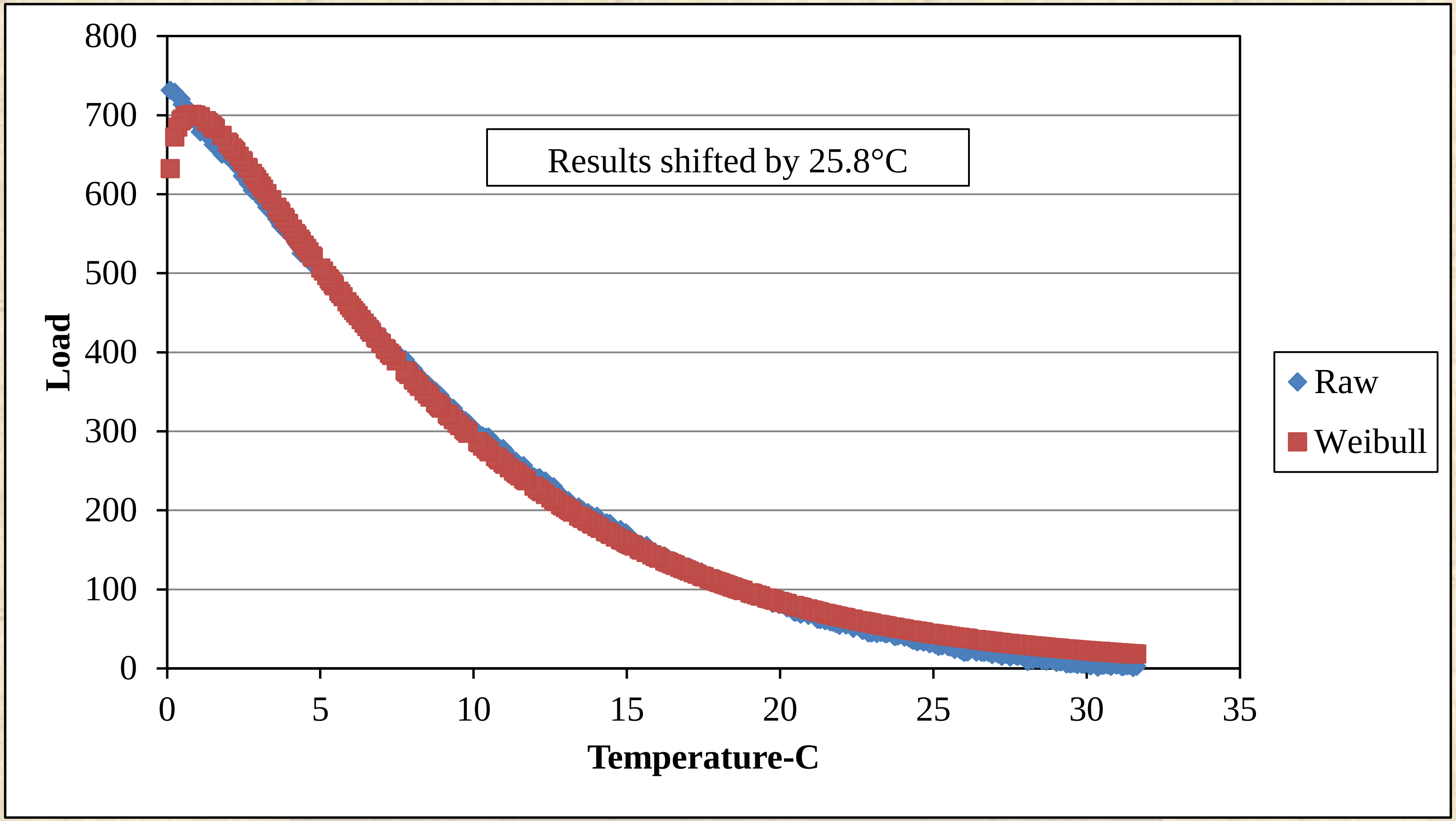
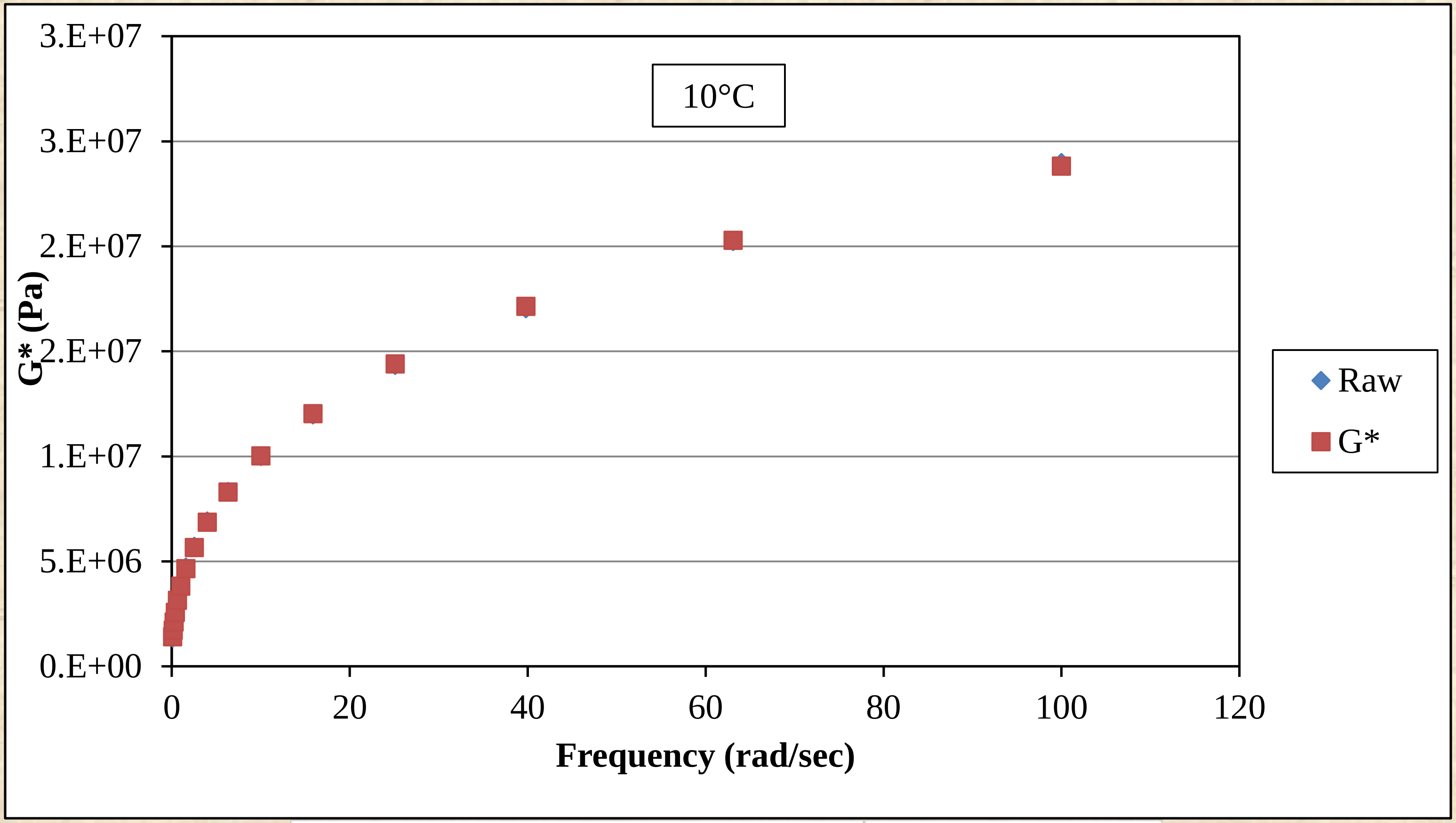
What is Next?

- TSRST



What is Next?

- G* at multiple frequencies



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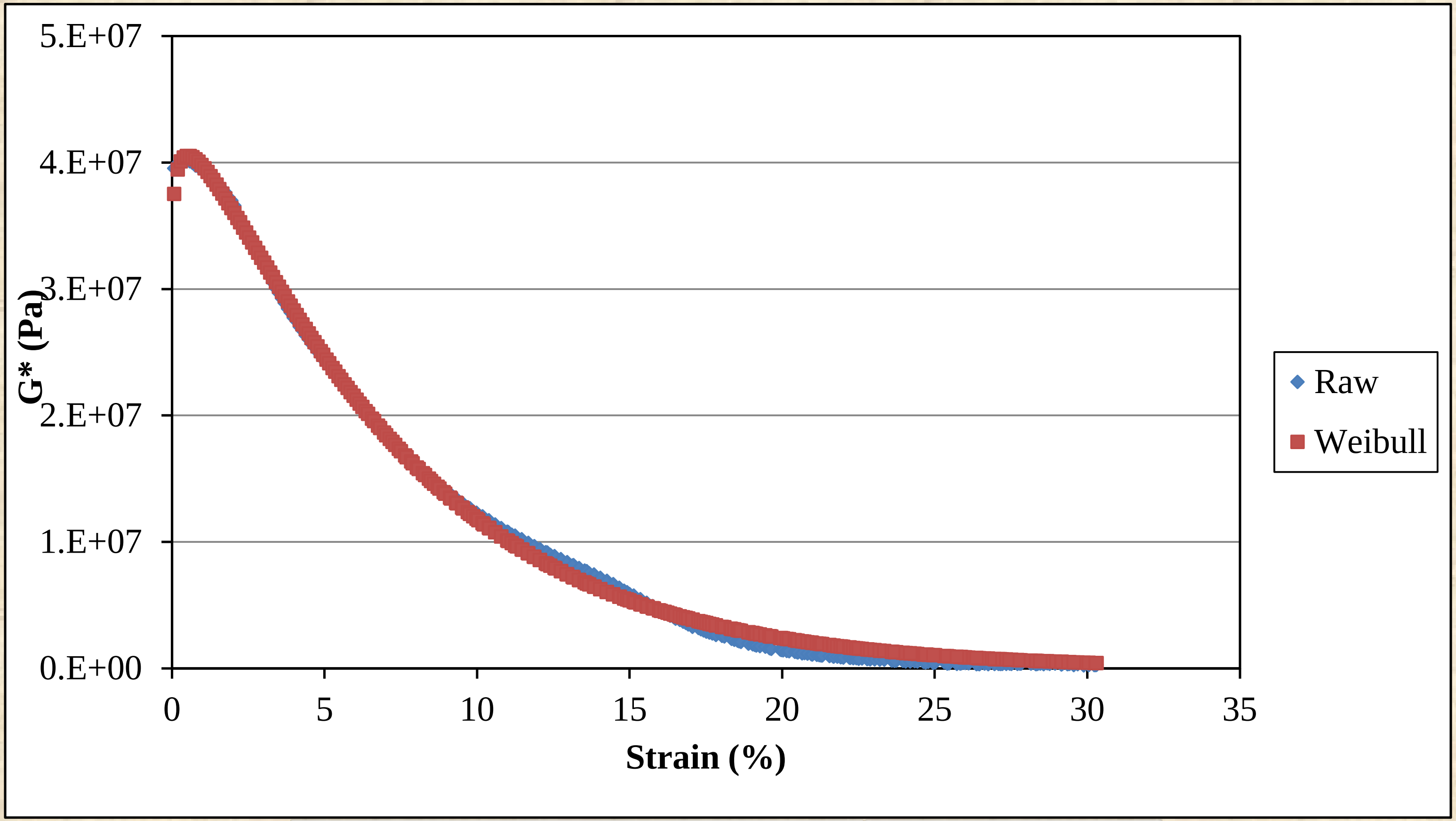
What is Next?

- LAS



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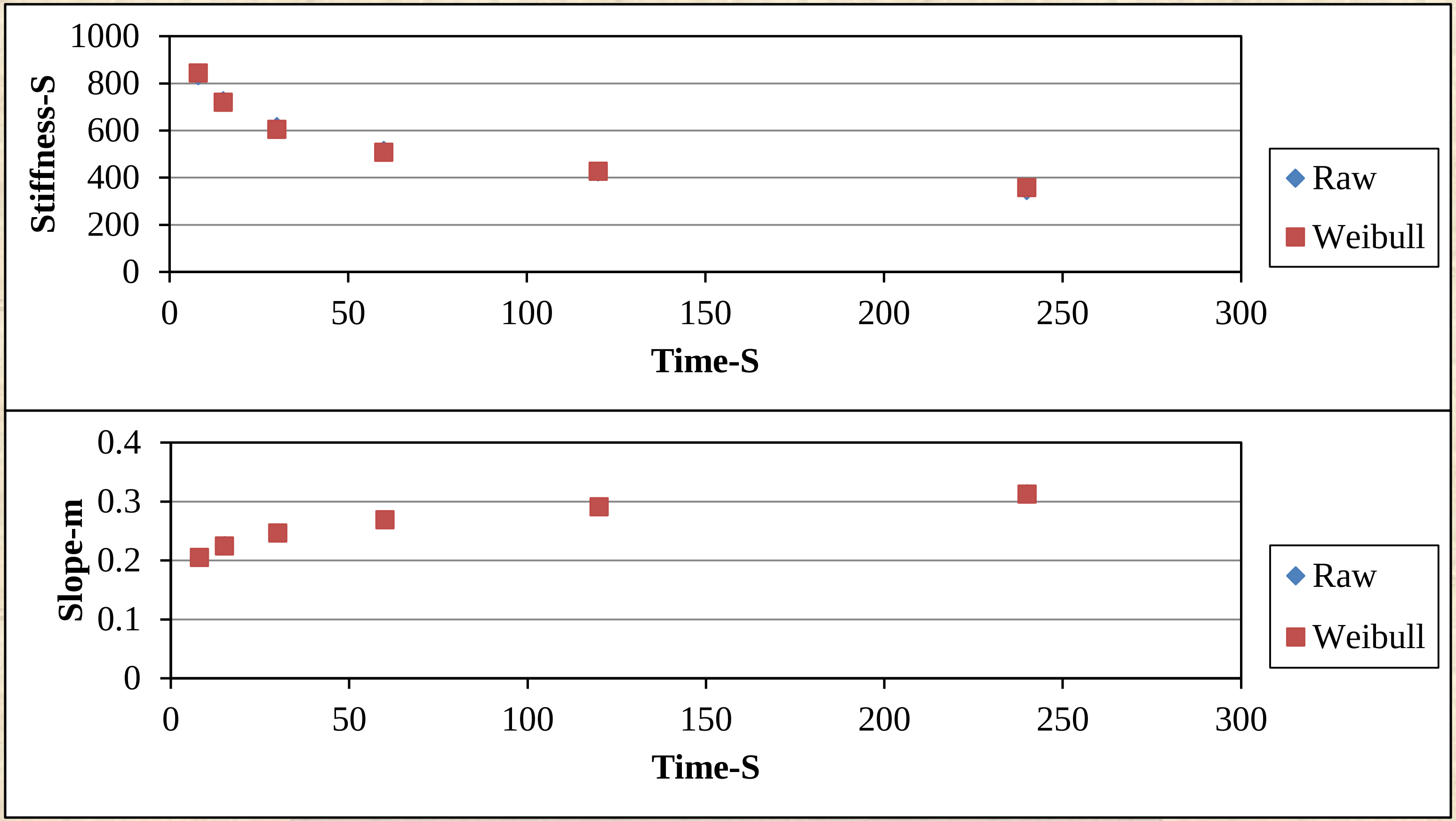
What is Next?

- BBR



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