Characterizing Asphalt Mixtures Resistance to Crack Propagation Using the SCB Test:

Weibull Distribution and Entropy Approach

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- The main idea of this research came up after a conversation with Professor Donald Christensen about Weibull Distribution.

- I would like also to Thank my advisors:
  
  Professor Walaa Mogawer  
  Professor Donald Christensen  
  Professor Ramon Bonaquist
Outline

- Introduction and Problem Statement
- Research Objectives
- Fitting Weibull Distribution to SCB Data
- Initial Complex Stiffness Modulus & Shannon Entropy
- Materials and Mix Design
- SCB Testing and New Approach to Analyze the Data
- Validating the New Approach
- Conclusions
Introduction and Problem Statement

SCB Test at Intermediate temperature

\[ J_c = \left( \frac{-1}{b} \right) \left( \frac{dU}{da} \right) \]

- \( J_c \) = critical value of the fracture resistance
- \( b \) = sample thickness
- \( a \) = the notch depth
- \( U \) = the strain energy to failure

\[ FI = A \frac{G_f}{|m|} \]

- Empirical

- Dissipated energy for asphalt mixtures as viscoelastic materials is unknown during loading.
- Amount of dissipated energy changes by changing temperature and it is not constant for all mixtures.
Weibull Distribution

Probability Density Function (PDF)

\[ f(x) = \left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^\beta} \frac{\beta}{\eta} \]

Cumulative Density Function (CDF)

\[ P(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^\beta} \]

Area under the curve = 1

\[ x > 0 \]

\[ X > 0 \]
Research Objectives

1) Fitting Weibull distribution to the relationship between load and load line displacement for SCB test.
2) Deriving mathematical equation for initial complex stiffness modulus of asphalt mixtures “Zo”.
3) Deriving mathematical equation for Shannon entropy “H” (A parameter that represents the mechanical behavior of asphalt mixtures).
4) Develop a new approach to characterize crack propagation resistance of asphalt mixtures using the SCB test.
Fitting Weibull Distribution to SCB Data

\[ F = W \left( \frac{x}{\eta} \right)^{\beta-1} e^{- \left( \frac{x}{\eta} \right)^{\beta} \frac{\beta}{\eta}} \]

- **F** = load (kN),
- **W** = Work of fracture (Joules),
- **x** = Load line displacement (mm),
- **\( \beta \)** = Shape parameter,
- **\( \eta \)** = Scale parameter (mm)
Initial Complex Stiffness Modulus

\[ F = W \frac{(x/\eta)^{\beta-1}}{\beta} e^{-(x/\eta)^{\beta}} \frac{\beta}{\eta} \]

\[ \frac{F}{(x)^{\beta-1}} = \frac{W(1/\eta)^{\beta-1}}{\beta} e^{-(x/\eta)^{\beta}} \]

Out of more than 200 specimens, the average \( \beta \) value was 1.99 and the standard deviation was 0.24

\[ \frac{F}{x} = \left( \frac{W}{\eta} \right) e^{-(x/\eta)^{\beta}} \]

\[ \frac{(F/A)/(x/r_p)}{(x/r_p)} = \left( \frac{r_p}{A} \right) \left( \frac{W}{\eta} \right) e^{-(x/r_p)^{\beta}} \]

Complex Stiffness Modulus of Asphalt Mixtures under indirect tensile stress \((Z)\)

At \( x = 0 \), the initial modulus can be written as

\[ Z_0 = Wr_p \beta / A \eta^2 \]


Assume \( r_p = 1\% \) Notch
Introduction to Shannon Entropy

\[ Z = \left( \frac{r_p}{A} \right) \left( \frac{W}{\eta} \right) e^{-\left(\frac{x}{\eta}\right)^\beta} \frac{\beta}{\eta} \]

\[ Z_0 = \frac{W r_p \beta}{A \eta^2} \]

\[ \frac{Z}{Z_0} = e^{-\left(\frac{x}{\eta}\right)^\beta} \]

The probability that the material will experience total failure “Percentage of drop in value \((x)\) can be expressed as:

\[ P(x) = \left[ 1 - e^{-\left(\frac{x}{\eta}\right)^\beta} \right] \]

Cumulative Density function of Weibull Distribution \(\beta>1\)
Entropy & Shannon Entropy

- Entropy is a physical property that indicates the molecular state of a system (a measure of disorder).
- The mechanical (effective) work of a system is a function of its entropy and internal energy.
- In statistical mechanics, Shannon entropy can be used as an indication to the mechanical behavior of a system with unknown state.
- Physical entropy can be estimated from Shannon entropy by multiplying it by a constant.
We want to pick a random ball from each box and return it back for four times.

What is the probability that the choice will match what is in the box?

P = 1 * 1 * 1 * 1 = 1
P = 0.75 * 0.75 * 0.75 * 0.25 = 0.1
P = 0.5 * 0.5 * 0.5 * 0.5 = 0.06

Probability (Knowledge)

High
Medium
Low

Shannon Entropy

\[ H = - \sum P_i \log_b P_i \]

\[ H = -[1 \times \log_{10} 1] = 0 \]
\[ H = -[0.75 \times \log_{10} 0.75 + 0.25 \times \log_{10} 0.25] = 0.244 \]
\[ H = -[0.5 \times \log_{10} 0.5 + 0.5 \times \log_{10} 0.5] = 0.3 \]
Shannon Entropy

\[ f(\tilde{x}) = r_p f(\tilde{x}) \]
\[ H = -\int_{-\infty}^{\infty} f(\tilde{x}) \ln(f(\tilde{x})) d\tilde{x} \]
\[ \tilde{x} = x/r_p \]

Euler-Mascheroni constant (0.577) = \( \gamma \left( 1 - \frac{1}{\beta} \right) + \ln \left( \frac{\lambda}{\beta} \right) + 1 \)
\[ \lambda = \eta/r_p \]

Properties of Shannon Entropy:

- A unique number for each distribution (SCB sample) depending on \( \beta \) and \( \lambda \).
- This unique number can be used as an indication to the mechanical properties (viscoelasticity) of mixtures.
## Materials and Mix Design

<table>
<thead>
<tr>
<th>Sieve Size (mm)</th>
<th>12.5</th>
<th>9.5</th>
<th>4.75</th>
<th>2.36</th>
<th>1.18</th>
<th>0.6</th>
<th>0.3</th>
<th>0.15</th>
<th>0.075</th>
<th>Binder Content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Passing by Weight</td>
<td>100</td>
<td>98</td>
<td>85</td>
<td>58</td>
<td>42</td>
<td>27</td>
<td>15</td>
<td>9</td>
<td>6</td>
<td>6.5</td>
</tr>
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</table>

### Incorporated Binder

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>-15</th>
<th>-5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG64-22 (2 Sources)</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>PG 58-28 (2 Sources)</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
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<td>PG 64-28</td>
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<td>✔</td>
<td>✔</td>
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<td>HiMA</td>
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</tr>
<tr>
<td>Formulated PG58-28</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
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<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>
It is necessary to change testing temperature based on the used binder

This plot helps to choose appropriate mixture based on the placement region

Max $R^2 = 0.99$
Min $R^2 = 0.96$
SCB Testing and Analysis

This plot helps to choose appropriate mixture based on the placement region.

Truncate data points beyond the peak Shannon Entropy.

Failure due to indirect tension

Failure due to shear

Max $R^2 = 0.96$

Min $R^2 = 0.81$

Formulated PG58-28
SCB Testing and Analysis

Max $R^2 = 0.97$
Min $R^2 = 0.88$
Validating the New Methodology

**Binder Content**

- $R^2 = 0.9694$
- $R^2 = 0.9667$
- $R^2 = 0.9287$
- $R^2 = 0.9651$

![Graph showing Zo (N/mm²) vs Temperature °C for different binder contents: PG64-22 Source C, PG64-22 Source C-0.4%, PG64-22 Source C+0.4%, and PG64-22 Source C+1% with $R^2$ values for each curve.]
Validating the New Methodology

- Binder Content

- Shannon Entropy-H

Temperature °C

R² = 0.9729
R² = 0.9196
R² = 0.9421
R² = 0.9587

PG64-22 Source C
PG64-22 Source C-0.4%
PG64-22 Source C+0.4%
PG64-22 Source C+1%
Validating the New Methodology

- Binder Content

![Graph showing the relationship between Shannon Entropy-H and Zo (N/mm²) for different PG64-22 sources with varying binder content. The graph includes lines for PG64-22 Source C, PG64-22 Source C-0.4%, PG64-22 Source C+0.4%, and PG64-22 Source C+1%. The R² values for each line are indicated: R² = 0.8849 for PG64-22 Source C, R² = 0.9398 for PG64-22 Source C-0.4%, R² = 0.9383 for PG64-22 Source C+0.4%, and R² = 0.945 for PG64-22 Source C+1%.]
Validating the New Methodology

- RAP Content

![Graph showing temperature vs. Zo (N/mm²) with lines for PG64-22 Source C, PG64-22 Source C+50% RAP. R² values: 0.9151 and 0.9694.]
Validating the New Methodology

**RAP Content**

![Graph showing Shannon Entropy vs. Temperature](attachment:image.png)

- **PG64-22 Source C**
  - R² = 0.9196

- **PG64-22 Source C+50% RAP**
  - R² = 0.9368
Validating the New Methodology

**RAP Content**

![Graph showing R² values and Shannon Entropy-H for PG64-22 Source C and PG64-22 Source C+50% RAP.]

- R² = 0.897
- R² = 0.8849
Validating the New Methodology

**NCAT ALF Mix**

<table>
<thead>
<tr>
<th>Zo (N/mm²)</th>
<th>Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

- R² = 0.8454
- R² = 0.8081
- R² = 0.8815
- R² = 0.8699
Validating the New Methodology

- NCAT ALF Mix

\[
\begin{align*}
R^2 &= 0.9019 \\
R^2 &= 0.7234 \\
R^2 &= 0.7671 \\
R^2 &= 0.7433
\end{align*}
\]
Validating the New Methodology

- NCAT ALF Mix

![Graph showing regression lines with R² values]
Validating the New Methodology

- NCAT ALF Mix
Conclusions

- Weibull distribution can be used to fit crack propagation data from the SCB test.
- The initial complex stiffness modulus and Shannon entropy (a measure of the mechanical behavior) can be derived from the Weibull fitted distributions.
- Correlations between testing temperature and initial complex stiffness modulus and Shannon entropy are useful to choose appropriate mixture based on the placement region.
- Based on this study, asphalt mixtures should be compared at the same state (Shannon entropy value), or at the same initial complex stiffness modulus. This might require testing at multiple temperatures.
THANK YOU!
SCB Testing and Analysis

Graphs showing load-line displacement (x) in mm for different temperatures (25C, 35C, 45C, 55C) with two different functions f(x) and P(x).
What is Next?

- Initial complex stiffness modulus and Shannon entropy can be predicted for other mixture and binder tests using similar approach.
- Initial complex stiffness modulus from different tests can be correlated using basics mechanics of materials.
- Shannon entropy from different tests might be correlated.
- A master curve can be developed from different tests with different modes of failures and used in pavement design.
What is Next?

❖ Beam Fatigue

[Graph showing the reduction of flexural stiffness over cycle number, comparing raw and Weibull data.]
What is Next?

Texas Overlay

![Graph showing Max Load/Cycle (lb) vs Cycle No. with two lines: Raw and Weibull]
What is Next?

❖ Cyclic tension (Pull-Pull)
What is Next?

Flow Number

![Graph showing MicroStrain vs Cycle No. with two curves: Raw and Weibull.](image-url)
What is Next?

**HWTD**

![Graph showing rut depth vs. number of passes]

- **Y-axis:** Rut depth (mm)
- **X-axis:** No. Of Passes

Legend:
- **Blue diamonds:** Raw
- **Red line:** Weibull
What is Next?

- TSRST

Results shifted by 25.8°C
What is Next?

- G* at multiple frequencies

![Graph showing G* at multiple frequencies](chart.png)
What is Next?

- LAS

![Graph showing G* (Pa) vs Strain (%)]

- Raw
- Weibull
What is Next?

**BBR**

![Graph showing stiffness and slope over time with two data sets: Raw and Weibull.](image)

- Stiffness-S vs. Time-S
- Slope-m vs. Time-S

Legend:
- **Raw**
- **Weibull**